ECN 275/375 Environmental and natural resource economics
Exercise set 5 – Eirik's suggested answers

Exercise 5.1 – Resource allocation mechanisms

Resource allocation mechanisms (RAMs) are the modern variant of principal-agent models.

(a) Write the three necessary criteria for a RAM to yield a predictable outcome, and explain what the three criteria are.

Answer: Informational viability, the participation constraint (in parts of the literature referred to as *individual rationality*), and incentive compatibility.

Informational viability: That the mechanism does not require more information than what is available. If not met, the regulator is not certain if what he/she believes are agents' actions actually take place.

The participation constraint: That it is in the agents' own self interest to participate. If not met, the expected turn-out (fraction of agents who participate)

Incentive compatibility: That it is in the agents' self interest to behave as expected. If not met, the regulation is unlikely to produce the expected results (outcomes).

(b) Why is incentive compatibility (IC) and Pareto optimality (PO) necessarily not jointly achievable?

Answer: Securing IC (or any of the two other necessary criteria for a predictable outcome) seldom comes for free. That implies some resources (funds) are needed to produce the incentives needed. That implies that as long as there are some costs associated with the policy, it cannot be Pareto optimal as these costs implies we may not be at the frontier of the production possibility set.

(c) Which is most important – incentive compatibility or Pareto optimality. Explain your answer.

Answer: With the above (a) definition/justification of incentive compatibility, the regulator cannot guarantee the expected outcome. Without certainty on the equilibrium we are in, welfare comparisons become meaningless:

Exercise 5.2 – Cost effectiveness and optimality in emissions space

Emissions space implies that the polluter's choice variable is emissions.

(a) Write down the mathematical definition for cost effectiveness emissions space. Explain the terms in the definition.

Answer: $MAC_i(m_i')=MAC_j(m_j')\forall i,j\in I$ (where m_i' is agent *i*'s chosen emission level, $MAC_i(m_i)$ denotes agent *i*'s marginal abatement costs of reducing emissions, m_i.

Verbal definition: All agents have equal marginal abatement costs of emissions evaluated at the agent's chosen emission level.

(b) Write down the mathematical definition for social optimality (efficiency). Explain the terms in the definition, and write the verbal definition for optimality.

Answer: $MAC_i(m_i^*)=MAC_j(m_j^*)=MD(M^*)\,\forall\, i,j\in I$ (where m_i^* is the (socially) optimal emission level for polluter i, $MAC_i(m_i)$ is the marginal abatement costs for polluter i in as a function of polluter i's emissions m_i, and $M^*=\sum_{k=1}^{K} m_k^*$ defines optimal aggregate emissions.

Verbal definition: The marginal abatement costs for all polluting agents at their (socially) optimal emission level is equal, and these marginal abatement costs also equals the marginal damages evaluated at the aggregate optimal emission level.

(c) Why is cost effectiveness necessary for optimality?

Answer: Without cost effectiveness, more resources (funds) than what is needed are spent on securing the desired outcome. Suppose there exists some other policy producing the same outcome using less resources. Then, the saved funds can be used to increase the welfare of at least one person in the economy, which means the cost ineffective policy cannot be welfare maximizing.

Remark: Being exact on notation important here to show understanding of central terms.

Exercise 5.3 – Cost effectiveness and optimality in public goods space

Emissions space implies that the polluter's choice variable in public goods space, q.

(a) Write down the mathematical definition for cost effectiveness public goods space. Explain the terms in the definition.

Answer: $MC_i(q_i')=MC_j(q_j')\,\forall\, i,j\in I$ (where q_i' is agent i's chosen supply level of the public good.

Verbal definition: All agents have equal marginal costs of producing the public evaluated at the agent's chosen level of public goods production.

(b) Write down the mathematical definition for social optimality (efficiency). Explain the terms in the definition, and write the verbal definition for optimality.

Answer: $MC_i(q_i^*)=MC_j(q_j^*)=MB(Q^*)\,\forall\, i,j\in I$ (where q_i^* is the (socially) optimal supply of q_i for polluter i, and $MC_i(q_i)$ is the marginal costs for agent i's production of q_i, and $Q^*=\sum_{k=1}^{K} q_k^*$ defines optimal supply of the good in question. **Remark:** If Q is a public good, summation is vertical.

Verbal definition: The marginal costs for all agents at their (socially) optimal supply level is equal, and these marginal costs also equal the marginal benefits evaluated at the aggregate optimal supply level.

Exercise 5.4 – Graphical demonstration that fixed emission permits may not be cost effective while tradable permits are

(a) Draw a graph showing why non-tradable (fixed) emission permits in general are not cost effective.

Answer: See the lecture note for this session

(b) Under what conditions would fixed permits be cost effective? Why is this an unlikely situation?

Answer: If the fixed permits exactly match the resulting tradable permit equilibrium. This is unlikely because it requires that the regulator has complete knowledge of all firms' marginal abatement cost functions, which usually in pollution economics are firms' private knowledge.

Exercise 5.5 – Emission constraints and Lagrange

In the following sub-questions $0<m_i<\bar{m}_i$ is the emission level for agent *i*, $\bar{m}_i \leq m_o^0$ is agent *i*'s maximum allowed emissions, and $\bar{M}$ is aggregate emissions. Note that there are $I \geq 2$ agents.

(a) Set up the equations needed for the fixed permits (non-tradable) case, i.e. $m_i \leq \bar{m}_i < m_i^0$, where formulate the Lagrangian, and comment on the cost effectiveness of the solution (you do not need to solve the problem).

Answer: $\underset{\{m_i\}}{\text{MIN}}\; TAC_i(m_i)$ s.t.$^I\, m_i \leq \bar{M}_i \,\forall\, i \in I$

The Lagrangian: $\mathscr{L}_i = TAC_i(m_i) + \lambda_i(m_i \leq \bar{m}_i) \,\forall\, i \in I$

Comment: This gives *I* Lagrangian problems, each with its own Lagrangian multiplier, λ_i . Given that marginal abatement costs increase with decreasing emissions m_i *,* the solution becomes $\lambda_i = MAC_i(\bar{m}_i)$ when agents have different marginal abatement costs functions.

Suppose that the principal (the regulator) knew the MAC-functions of all agents and hence could set agent's individual permits (quotas) so that $MAC_i(\bar{m}_i) = MAC_k(\bar{m}_k) \,\forall\, i,k \in I$. This conflicts with the private knowledge requirement.

(b) Set up the equation needed for the tradable permits problem and formulate the Lagrangian for the problem where aggregate emissions are less than or equal to the aggregate emission target.

Answer: $\underset{\{m_i\}}{\text{MIN}} \sum_{i=1}^{I} TAC_i(m_i)$ s.t. $\sum_{i=1}^{I} m_i \leq \bar{M}$

The Lagrangian: $\mathscr{L} = \sum_{i=1}^{I} TAC_i(m_i) + \lambda \sum_{i=1}^{I} m_i \leq \bar{M}$

Comment: This gives *one* Lagrangian where the aim is to minimize total costs for the sum of the *I* agents. Given that marginal abatement costs increase with decreasing emissions m_i the solution becomes $\lambda = MAC_i(m_i^C) = p_M \,\forall\, 0 < m_i^C < m_i^0$ as agents buy or sell permits until their marginal abatement costs evaluated at their individually chosen emission level m_i^C , equals the permit price p_M .

An exception would be corner solutions which is excluded due to the condition

Exercise 5.6 – Bath tub diagram where the resulting quota price is zero

Draw a bath tub diagram showing a situation where the tradable permit price is zero.

Answer: Make the "bath tub" so wide that the two marginal abatement cost functions do not intersect. Then, the resulting permit market price becomes zero. This entails a non-binding emission constraint, i.e. a corner solution.

Exercise 5.7 – Bath tub diagram for two sectors – the optimal solution

The required total reduction in emissions equals z_{A+B} = 100 for two sectors such that $z_A + z_B$ = 100.

Sector A's marginal cost function of supplying emissions reductions is $MC_A(z_A) = z_A$, while sector B's marginal cost function is $MC_B(z_B) = z_B/3$.

(a) Which of the two sectors do you expect needs to reduce emissions the most for a least cost (cost effective) distribution of emissions reductions? Briefly explain why.

Answer: Sector B, because it has the lowest marginal abatement costs (increases by 1/3 compared to sector A.

(b) Find the least cost distribution of emissions reductions for the two sectors with a bath tub diagram.

Answer: Draw a bathtub that is 100 wide, and draw A's marginal cost function from one side, and B's marginal cost function from the other side. As cost effectiveness (the least cost solution requires) the marginal costs to be equal at the optimal distribution, the optimal solution is where the two curves cross.

(c) Solve mathematically for the optimal distribution of emissions reductions supplied for A and B. What is the marginal costs at the optimal distribution?

Answer: This can be done using a Lagrangian or by substitution. The latter is the easiest: As total emissions reductions must equal $z_{A+B} = 100$, you can set one sector's emissions reductions as 100 less the other sector's reductions. This gives $z_B = 100 - z_A$ (or $z_A = 100 - z_B$).

Keeping z_B gives: $MC_A(z_A) = MC_A(100 - z_B) = 100 - z_B = z_B/3 = MC_B(z_B)$ which simplifies to

$100 = z_B/3 + z_B = 4z_B/3 \Rightarrow z_B^* = 3/4(100) = 75 \Rightarrow z_A^* = 25$.

The marginal costs equal 25 (follows directly by inserting $z_A^* = 25$ in the MC-function for A, and checks out correctly if you insert $z_B^* = 75$ in the MC-function for B.

Remark: If you choose to solve this problem using a Lagrangian approach, remember that you already has the marginal cost functions given. One way to ensure you do not mix things up, set up the normal constrained optimization problem:

$$\underset{\{z_A z_B\}}{\text{MIN}}\ TC_A(z_A) + TC_B(z_B) \quad \text{subject to } (z_A + z_B = 100)$$

which gives the Lagrangian: $\mathscr{L} = TC_A(z_A) + TC_B(z_B) + \lambda(100 - z_A - z_B)$ (note how the constraint is written to get the right signs), which gives the following 3 first order conditions:

(1) $\dfrac{\partial \mathscr{L}}{\partial z_A} = \dfrac{\partial TC_A}{\partial z_A} - \lambda = MC_A - \lambda = z_A - \lambda = 0$

(2) $\dfrac{\partial \mathscr{L}}{\partial z_B} = \dfrac{\partial TC_B}{\partial z_B} - \lambda = MC_B - \lambda = z_B/3 - \lambda = 0$

(3) $\dfrac{\partial \mathscr{L}}{\partial \lambda} = 100 - z_A - z_B = 0$

Solving this gives the same cost effective solution as before, i.e. $z_A^C = 25$ and $z_B^C = 75$ plus the solution for $\lambda^C = 25$, which is the marginal cost of the aggregate public good supply constraint.