Rational expectations
– lecture note in macroeconomics

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This notes presents the basics of rational expectations theory in macroeconomis. Its key point is that rule based monetary policy where central banks make the rule they use for setting monetary policy, particularly the steering rate, *i*, enable economic agents to make choices that are more in line with their preferences. All other things equal, this may increase societal welfare in an economy.

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1. Background on choice under uncertainty

Expected utility theory (von Neumann and Morgenstern, 1944) enabled the use of certainty equivalents for assessing choice under uncertainty. Von Neumann-Morgenstern expected utility (VNM-EUT) has since come under scrutiny. Notable contributions in this regard include:

- Allais (1953), who found that participants in economic experiments frequently violated the certainty equivalence aspect from expected utility theory. This violation has later been known as the Allais paradox, which contributed to Allais receiving the Nobel memorial prize in economic science in 1988.
- Kahneman and Tversky (1979) looked at the asymmetry between losses and gains in what they termed prospect theory. Ensuing works on violations of VNM-EUT axioms have been important for the development of behavioral and experimental economics. Kahneman was central in many of these works, for which he received the Nobel memorial price in economic science in 2002 (shared with Vernon L. Smith).

Simon (1956) viewed VNM-EUT far more promising than for example Allais. In Simon (ibid.) he showed that under quadratic cost functions and some simplifying assumptions about the nature of uncertainty function, firms could maximize expected profits using certainty equivalence. This work is along the lines Markowitz (1952), the foundation for mean-variance method. Müller and Machina (1987) showed that the mean-variance approach gives the same ranking of risky alternatives as VNM-EUT when the objective function is quadratic and the distribution of risky outcomes can be fully described by the two first moments (the mean and variance) of the probability distribution. My interpretation is that with these strong limitations on preferences and settings, little is gained using the mean-variance method over the VNM-EUT.

Nerlove's famous paper on hog price cycles (1958) explained how replacing naive expectations, i.e. prices the next time period equal current prices, with adaptive expectations could solve this issue.

Adaptive expectations analyses use a series of past prices to predict future prices more accurately using higher order auto-regression procedures (AR-*n*, where *n* is the number of lagged time time periods of the dependent variable)[1]. This forecasting approach broke the cobweb pattern emerging from naive expectations.

2. The foundations of rational expectations

Muth (1961) marks the start of the rational expectations literature. He was intrigued by Nerlove (1958) when he sought to develop a universally robust theory for understanding market impacts from the actions of individual agents. Summarizing past findings he wrote:

> *To make dynamic economic models complete, various expectational formulas have been used. There is however little evidence to suggest that the presumed relations bear a resemblance to the way the economy works.* (Muth, 1961:315).

Muth (ibid.) then presents the intuition behind the term *rational expectations*:

> *I should like to suggest that expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory. At the risk of confusing this purely descriptive hypothesis with a pronouncement as to what firms ought to do, we call such expectations "rational".* (Muth, ibid.: 316).

To make the term *rational expectations* operational, Muth introduced the expected conditional expectations given the most recent available information, I_{t-1}, which are given by:

$$E(X_t \mid I_{t-1}) = \int_a^b X_t f(X_t \mid I_{t-1})\, dX_t \tag{1}$$

where X_t is a continuous random variable[2] with lower and upper bounds *a* and *b* respectively. The forecast error then becomes:

$$\varepsilon_t = X_t - E(X_t \mid I_{t-1}) \tag{2}$$

At time $t-1$ the conditional expectation is known. Hence, the conditional expectation equals the forecast. We get:

$$E(\varepsilon_t \mid I_{t-1}) = E(X_t \mid I_{t-1}) - E(X_t \mid I_{t-1}) = 0 \tag{3}$$

Equation (3) implies that the rational expectations should yield expected forecast errors equal to zero, i.e. they are unbiased.

Returning to Muth's (1961:316) verbal formulation, it follows that all available relevant information (in line with economic theory) should now be utilized. This implies that the probability distribution of the forecast errors should have the minimum attainable variance,

$$\sigma_t = var(\epsilon_t) \leq var(\xi_t) \tag{4}$$

where ξ_t is the forecast error of any alternative model. This requires that all available and relevant information in the forecast errors, ϵ_t, is effectively utilized, i.e. there is no systematic patterns in the forecast errors of a model founded on rational expectations. Here, we note that this does not imply perfect knowledge, only that information which is available or can profitably collected, is used. An implication of "profitably used" is that economic agents only will collect information to the point where it pays to do so.

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1 Example AR-2: $P_t = \alpha + \beta_1 P_{t-1} + beta_2 P_{t-2} + \boldsymbol{\gamma X}$ where P_t is the dependent variable (often prices) at *t* is a discrete time index, α is a constant term, β_1 and β_2 are parameters of the lagged values of the dependent variables, and $\boldsymbol{\gamma}$ is a vector of parameters for other exongeous (independet variables) $\boldsymbol{X}_t$ at time *t*.

2 A discrete form also exists, but here it suffices to work with the continuous form.

The key takeaways from this sub-section are that rational expectations lead to:

- Unbiased forecast errors, i.e. the expected value of the forecast errors are zero (equation 3).
- That the minimum attainable variance in the forecast errors is reached (equation 4).

3. Monetary policy and rational expectations

Friedman (1968) saw rational expectations as particularly important for the impacts of monetary policy, which is closely linked to interest rates. As central banks' steering rates affect interest rates, the steering rate is highly relevant . His 1968 paper set the tone for ensuing works in macroeconomics also beyond steering rates. Besides steering rates, monetary policy also involves the money supply which today is frequently adjusted using governmental bonds: When agents buy money market bonds that reduces liquidity which implies that interest rates increase. In turn, that leads to reduced consumption and a subsequent economic "cooling off". Conversely, when a central bank buys back money market bonds, liquidity increases and consumption grows, all other things equal.

A premise in Muth (1961) is that economic agents use relevant and valid models of the economy. One way of increasing agents' use of such models is that central banks are open about the criteria they use when deciding monetary policy. This is a key message in the seminal paper "*Rules Rather than Discretion: The Inconsistency of Optimal Plans*" by Kydland and Prescott (1977).

Modern central banks therefore often publish the formulas and the key information they use when deciding monetary policy. A typical structural form expression for central banks' decisions is:

$$i = z^i + d_1(\pi - \pi^*) + d_2\frac{Y - Y^n}{Y^n}, \quad d_1 > 0, d_2 > 0 \tag{5}$$

where i denotes the nominal interest rate, z^i is an interest rate shock, π and π^* are the inflation rate and its target, and $(Y - Y^n)/Y^n$ is the GDP gap in percent. d_1 og d_2 are weighting factors for the deviations from the inflation target and the GDP gap. Using a set of typical equations to represent an economy (see Holden, 2016:234)[3] the reduced form becomes:

$$i = z^i + d_1(\pi^e - \pi^*) + d_1 z^\pi + (d_1\beta + d_2)\frac{Y - Y^n}{Y^n} \tag{6}$$

where $\beta > 0$ is a collection parameter for parameters in the structural equations, and z^π is the inflation shock variable.

Friedman (1968) also set the stage for what have been termed the "rational expectations revolution" in macroeconomics in the mid 1970s. Robert E. Lucas jr. and Thomas J. Sargent[4] were central in developing rational expectations theory. Early seminal works include Lucas (1973), and Sargent (1976).

4. Macroeconomic implications

Before the "rational expectations revolution" the Phillips curve (Phillips, 1958) was central for monetary policy. Under the Phillips curve, central banks can trade off inflation with higher unemployment and vice-versa. In a joint conference presentation Lucas and Sargent (1978) showed that under rational expectations, and flexible prices and wages, the Phillips curve would not hold. Their arguments resonated well with the stagflation in the late 1970s.

3 Holden (2016) is one of the most frequently used text books in intermediate macro economics courses at Norwegian universities.

4 Both received the Nobel Memorial Prize in Economic Science, Lucas in 1995, and Sargent in 2011 (shared with Christopher A. Sims).

Lucas and Sargent (ibid.) point to some works like Fischer (1977), that showed that under sticky wages, for example from wage negotiations giving tariff wages, the Phillips curve would work. They still maintained that rational expectations to a large extent made Keynesian fiscal policies obsolete.[5] Despite their view, Fishers' work became instrumental for what later was termed the Keynesian framework.

Radner (1979) proved the existence of a rational expectations equilibrium based on agents' subjective beliefs about future prices. A rational expectations equilibrium contains all relevant information agents need to make better informed decisions. This implies that agents learn and upgrade their beliefs producing more efficient market outcomes. An agent who does not learn is expected to do worse in the market and exit.

5. Estimation issues

Central banks operate with public decision rules like the one in equation (6) to set the steering rate, *i*. Ideally, the central bank should have perfect expectations equal to realized values. This is reflected in equation (7) where the time index *t*, and decay functions decay functions $f^{i}(z_t^{i})$ and $f^{\pi}(z_t^{\pi})$ on the (stochastic) shock variables with unknown magnitudes:

$$i_t = f^{i}(z_t^{i}) + d_1(\pi_t^{e} - \pi^{*}) + d_1 f^{\pi}(z_t^{\pi}) + (d_1\beta + d_2)\frac{Y_t - Y_t^{n}}{Y_t^{n}} \tag{7}$$

Central banks decide on the steering rate, i_{t+1} for the next time period $t+1$, usually next month, based on the available information they have at time *t* for the right hand side variables. Writing expectations for the next time period, $t+1$, based upon the available information at time *t*, takes the form ${}_t x_{t+1}^{e}$. This gives:

$$i_{t+1} = f^{i}(z_t^{i}) + d_1({}_t\pi_{t+1}^{e} - \pi^{*}) + d_1 f^{\pi}(z_t^{\pi}) + (d_1\beta + d_2)\frac{{}_t Y_{t+1}^{e} - Y_{t+1}^{n}}{Y_{t+1}^{n}} \tag{8}$$

where Y_{t+1}^{n} is the assessed normal capacity GDP. This is usually done by multiplying last month's GDP by $1+\gamma_t$ where γ_t is the long term monthly growth. Estimating (8) requires deciding what will be next months' expected inflation rate, ${}_t\pi_{t+1}^{e}$, and expected GDP, ${}_t Y_{t+1}^{e} = (1+\gamma_t) Y_t$.

Townsend (1978) showed how equations like (8) without the shock variables (red terms) can be estimated using subjective expectations for all expectation variables (those preceded with subscript *t*) and updated using Bayesian approaches. Note that this perspective is consistent with Radner (1979) on the existence of a rational expectations equilibria already mentioned.

Given the central banks' emphasis on openness in setting the steering rate, the short duration of a period and hence usually small change from one month to another, a more open procedure is to set the steering rate using last month's figures, i.e.:

$$i_{t+1} = f^{i}(z_t^{i}) + d_1(\pi_t - \pi^{*}) + d_1 f^{\pi}(z_t^{\pi}) + (d_1\beta + d_2)\frac{Y_t - Y^{n}}{Y_t^{n}} \tag{9}$$

This formulation does not fully utilize the information gains from moving to a rational expectations equilibrium, but all agents have access to the same information including the impacts of past unexpected shocks on the economy for setting the steering rate.

5 *Ricardian equivalence* (Barro 1979) became a far more important criticism of Keynesian counter cyclical fiscal policies. The main reason for this was that increased government spending to boost the economy when GDP was below the neutral GDP (Y^{n}) would have limited effects as consumers feared future tax increases to pay for the increased public debt. Barro (1989) gives a non-technical exposition of Ricardian equivalence.

6. Literature

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