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The Economics of Stock Pollutants

Lecture Notes

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Contents

Preface

This note is part of a planned series of lecture notes on dynamic resource economics. The notes are at this time incomplete and should be viewed as preliminary.[1] However, I would appreciate any comments and suggestions which the reader may have.[2]

1 Introduction

The standard textbook model of pollution is static and is concerned mainly with flow pollutants.

[1]These notes are intended as a help for my students, and are, for the time being, not for general distribution to a broader audience.

[2]Send e-mail to `olvar.bergland@ios.nlh.no`.

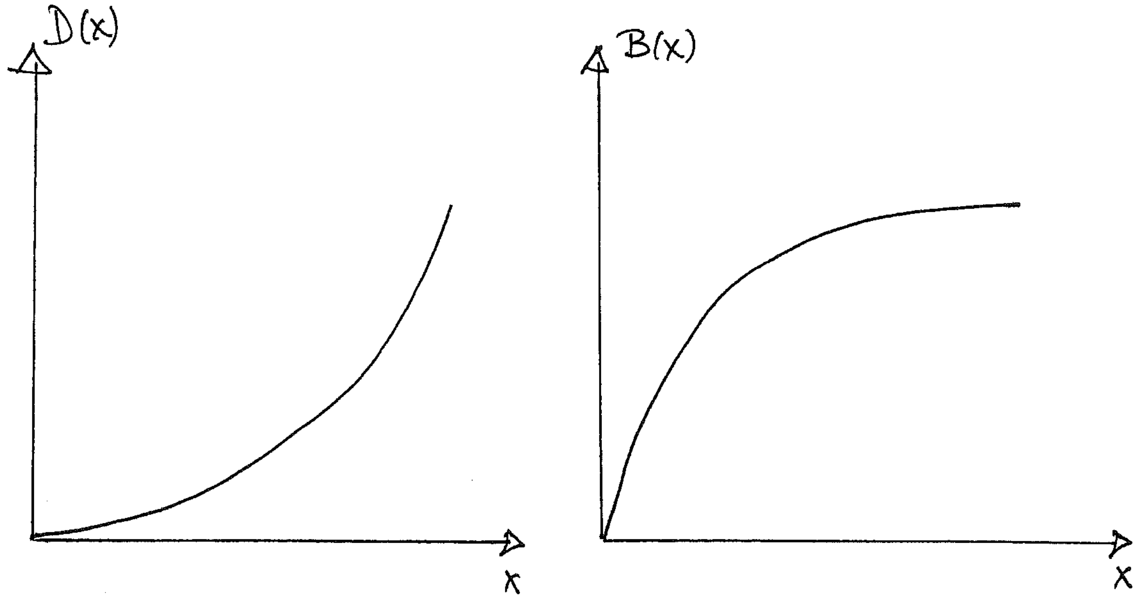

Figure 1: The benefit and damage functions from emissions.

The model ignores some important aspects of the behavior pollutants in the environment, i.e. some pollutants are pretty harmless in small concentrations, but they tend to accumulate and become toxic in higher concentrations. Others may be toxic today, but are broken down into harmless components rather quickly, while others may remain unchanged a long time to come.

The intuition is that the possibility of accumulation of pollutants in the natural environment should have implications for their release today.

These notes extends the standard economic model of optimal pollution to a dynamic setting where pollutants are accumulating and decaying over time.

2 Static Model

The static model of optimal pollution does not distinguish clearly between stocks and flows. In anticipation of later analysis let the static model to be concerned only with flows, hereafter referred to simply as emissions.

Let x be the emission (flow) of some pollutant. The damage inflicted on society is summarized with the *damage function* $D(x)$. The damage function is strictly increasing in x at an increasing rate, i.e. $D(x)$ is a convex function in x. The *benefit function*, $B(x)$, is strictly increasing in x, but at a decreasing rate, i.e. it is a concave function in x. The benefit and damage functions are shown in figure 1.

The standard formulation of the optimal pollution model is to find that emission level which maximizes the net benefits, i.e.

$$\max_{x} \quad \big(B(x) - D(x)\big) \tag{1}$$

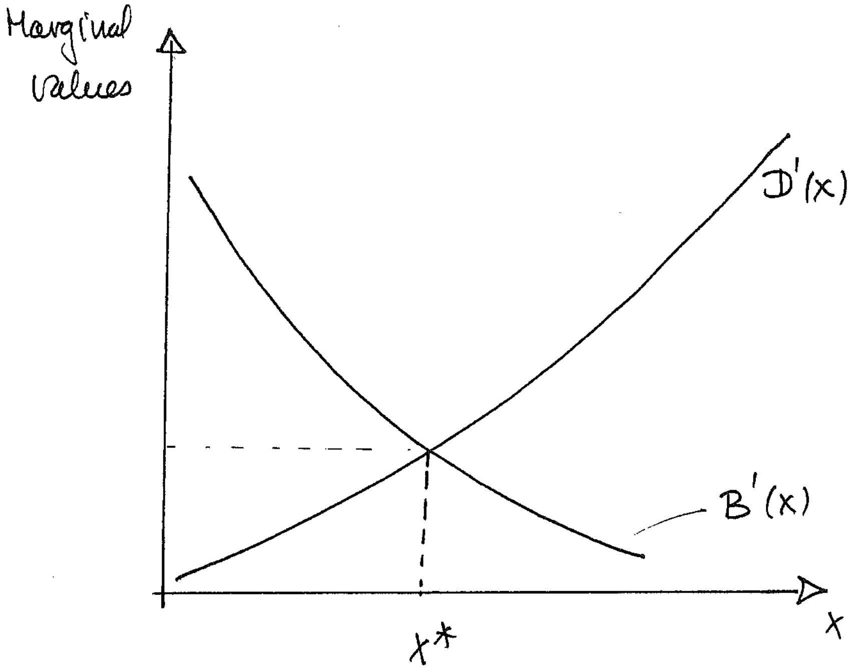

Figure 2: The determination of the optimal emission rate.

The first-order necessary conditions for this maximization problem is

$$B'(x) - D'(x) = 0.$$

Thus we have the standard result from environmental economics that at the optimal emission level do marginal benefits equate marginal damages:

$$\boxed{B'(x) = D'(x)} \tag{2}$$

This is illustrated in the by now familiar graph in figure 2, where the intersection of the marginal benefit curve with the marginal damage curve gives the optimal emission rate.

3 A Dynamic Model

3.1 Pollutant behavior

The question is now what happens with the emitted pollutant. If the pollutant is immediately broken into harmless substances this, is the end of the story. The other extreme possibility is that the pollutant is *perfectly persistent*, i.e. it is in no way broken down, dissolved or absorbed. In between is the case of a *decaying* pollutant. Given sufficient time, chemical and biological processes will transform the pollutant into less harmful substances (hopefully).

Let $S(t)$ denote the stock of the pollutant at time t, and consider the following linear decay equation

$$\dot{S}(t) = x(t) - \theta S(t) \tag{3}$$

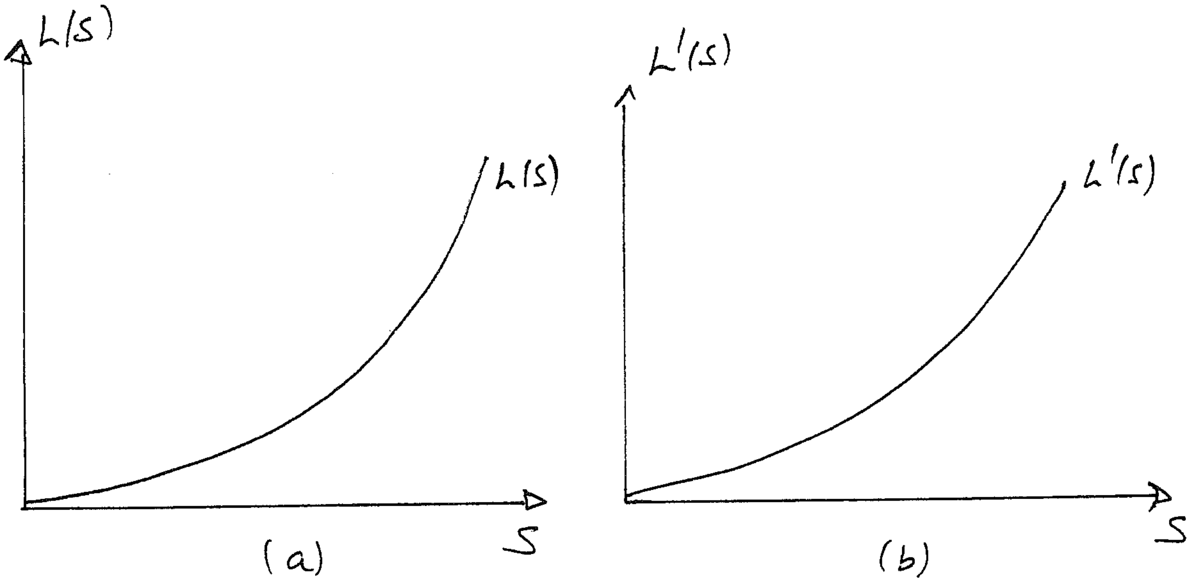

Figure 3: The damage function $L(S)$ (a) and the marginal damage function $L'(S)$ (b).

where $x(t)$ is the emissions at time t and θ is the decay constant.[3]

The equation gives the time behavior of the stock of the pollutant. The stock increases with increasing emissions, and is decaying with the rate θ.

As before let $B(x(t))$ be the benefit and $D(x(t))$ the damage arising from the emission rate $x(t)$. Let $L(S(t))$ be the damage caused by the stock of the pollutant, and where L is strictly increasing in S at an increasing rate. Both the damage function L and marginal damages $L'(S)$ are shown in figure 3.

3.2 Optimal control model

We can use our standard model formulation of a dynamic optimization problem to find the optimal emission level x over time. The objective is to maximize the present value of benefits less damages, i.e.

$$\max_{x(t)} \int_0^{\infty} \big(B(x(t)) - D(x(t)) - L(S(t))\big)e^{-rt}\, dt \tag{4}$$

subject to

$$\dot{S}(t) = x(t) - \theta S(t), \qquad S(0) = S_0. \tag{5}$$

The technique for solving this optimal control problem is first to create the current value Hamiltonian function

$$\mathcal{H} = B(x) - D(x) - L(S) + \pi(x - \theta S) \tag{6}$$

[3]Most likely will the decay rate depend on stock, and the "quality" of the environment and more complex models with non-linear decay functions are appropriate. I will use this simple model to avoid technical difficulties, but most of the conclusions are valid under less restrictive assumptions.

where π is the dynamic multiplier associated with the state variable.

From the maximum principle

$$\frac{\partial \mathcal{H}}{\partial x} = B'(x) - D'(x) + \pi = 0 \tag{7}$$

$$\dot{\pi} - r\pi = -\frac{\partial \mathcal{H}}{\partial S} = L'(S) + \pi\theta \tag{8}$$

$$\dot{S} = \frac{\partial \mathcal{H}}{\partial x} = x - \theta S \tag{9}$$

Equation (7) shows the linkage to the static model. Rewrite the equation as

$$B'(x(t)) = D'(x(t)) - \pi(t) \tag{10}$$

i.e. the marginal benefits of emission are at any point in time equal to the marginal damages of emissions less the intertemporal marginal value of stock pollutant (π).

4 Steady-state

We can explore the properties of this model by analyzing the steady-state behavior. Steady-state would require $\dot{\pi} = \dot{S} = 0$. Thus from equation (9),

$$\dot{S} = 0 \qquad \Rightarrow \qquad \boxed{x^* = \theta S^*} \tag{11}$$

In the steady-state optimal emissions are equal to the decay amount of the steady-state stock level of the pollutant.

Note that from equation (7)

$$\pi = D'(x) - B'(x) \tag{12}$$

and from equation (8)

$$\dot{\pi} = L'(S) + (r + \theta)\pi. \tag{13}$$

Setting $\dot{\pi} = 0$ and substituting in gives

$$\dot{\pi} = 0 \qquad \Rightarrow \qquad \boxed{B'(x^*) = D'(x^*) + \frac{L'(S^*)}{r + \theta}} \tag{14}$$

The interpretation of this condition is that in the steady-state are marginal benefit equal to marginal damage from emissions plus the present value of an infinite series of marginal damages caused by the stock pollutant (this is the "intertemporal marginal value of stock pollutant", i.e. π^*, as discussed above). This last component can be explained by noting that $L'(S^*)$ is the damage caused by marginally increasing the permanent level of stock, and thus dividing by the interest rate plus the decay rate gives the present value.

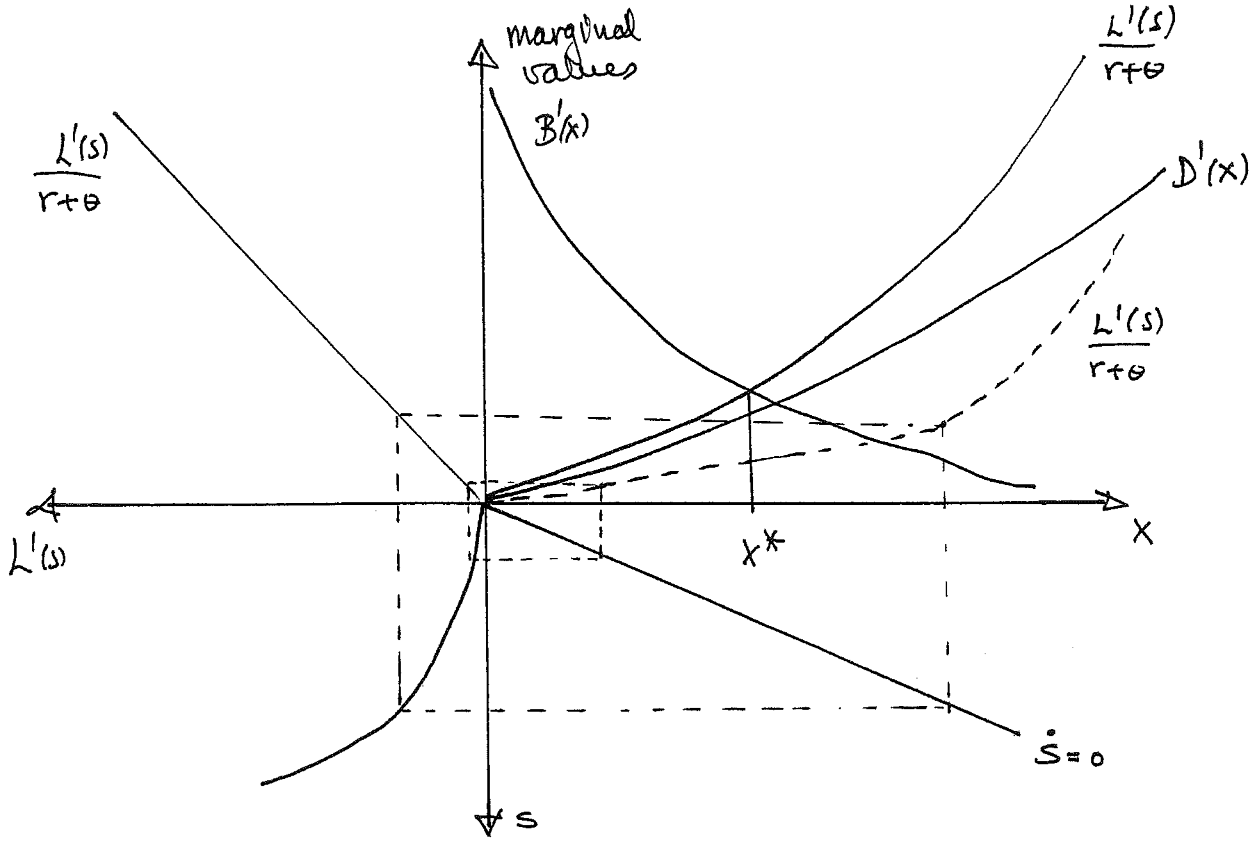

Figure 4: The determination of the optimal dynamic emission rate.

5 Graphical Illustration

To gain insight into the solution to the optimal emission problem some graphs may be useful.

5.1 Extending the static model

It is possible to amend the standard graphical analysis such that the stock pollutant is taken into account. The marginal condtions are given in equation (14). Although parts of that condition is well-known from the static analysis, the last term needs to be added. This can be done using the steady-state relationship between emissions and stocks (equation (11)).

The graph in figure 5 is constructed as follows:

1. In quadrant I draw the marginal benefit and damage curves for emissions. (This is the standard static analysis.) Their intersection identifies the optimal emission level ignoring any stock effects.

2. In quadrant IV draw the steady-state condition $\dot{S} = 0$, which in this case is a straight line given by $x = \theta S$.[4]

3. In quadrant III draw the marginal damage curve for the stock pollutant, i.e. $L'(S)$.

[4]Using a different decay function would alter this curve.

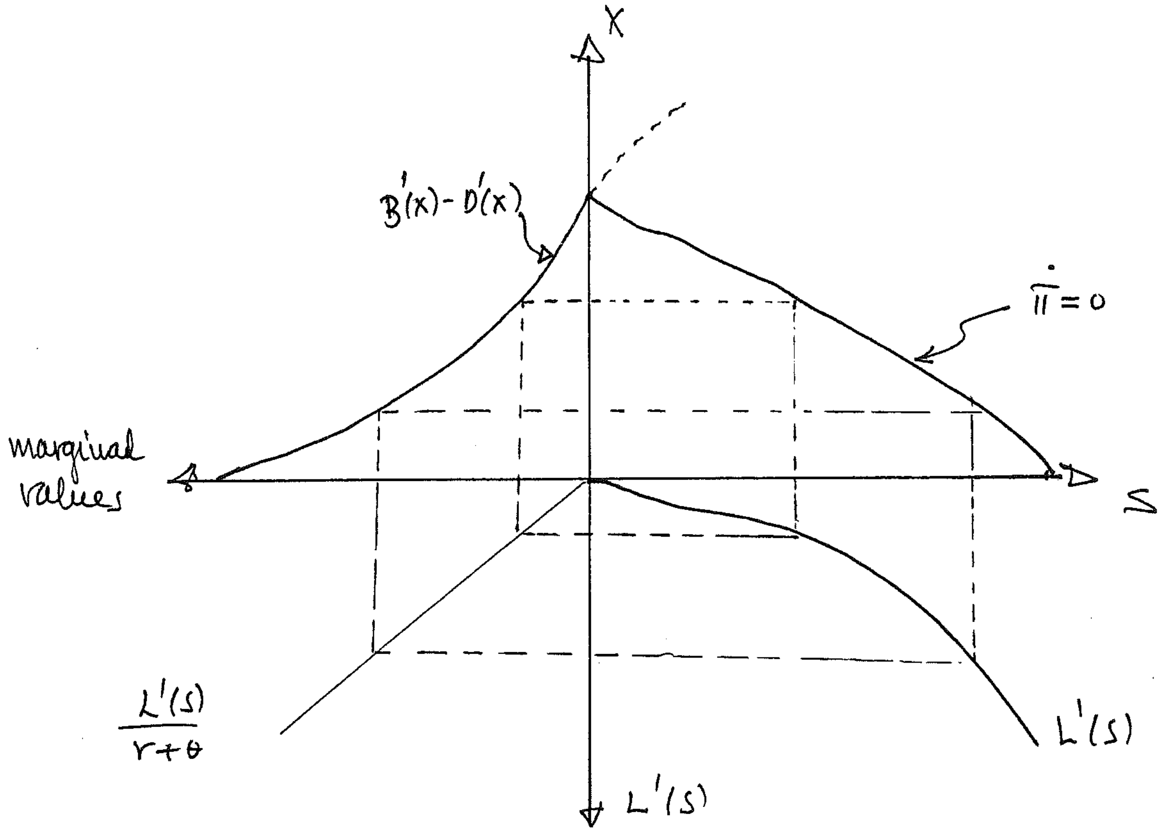

Figure 5: The determination of the $\dot{\pi} = 0$ curve.

4. Finally, I need to translate $L'(S)$ into present value terms. This is done by drawing the line

$$\frac{L'(S)}{r+\theta}$$

in quadrant II.

5. Start with some arbitrary emission level, say x, and find the corresponding stock level S, marginal damages $L'(S)$, and present value. This gives a single point in quadrant I. Repeating this step should trace the curve $'\frac{L'(S)}{r+\theta}$.

6. The final step is to add this curve to the marginal damage curve.

... and the optimal emission level with stock effects is now identified as x^*.

5.2 Phase-diagram

The dynamic behavior of this model can also be illustrated graphically. I need to be able to graph both the steady-state conditions. On one hand is the steady-state condition $\dot{S}(t) = 0$ a linear equation, see equation (11). However, for $\dot{\pi} = 0$ there is only an implicit equation in x and S, namely equation (14). Rewrite this as

$$B'(x) - D'(x) = \frac{L'(S)}{r+\theta}. \tag{15}$$

The graphical solution here is given in figure ??, and is obtain as follows:

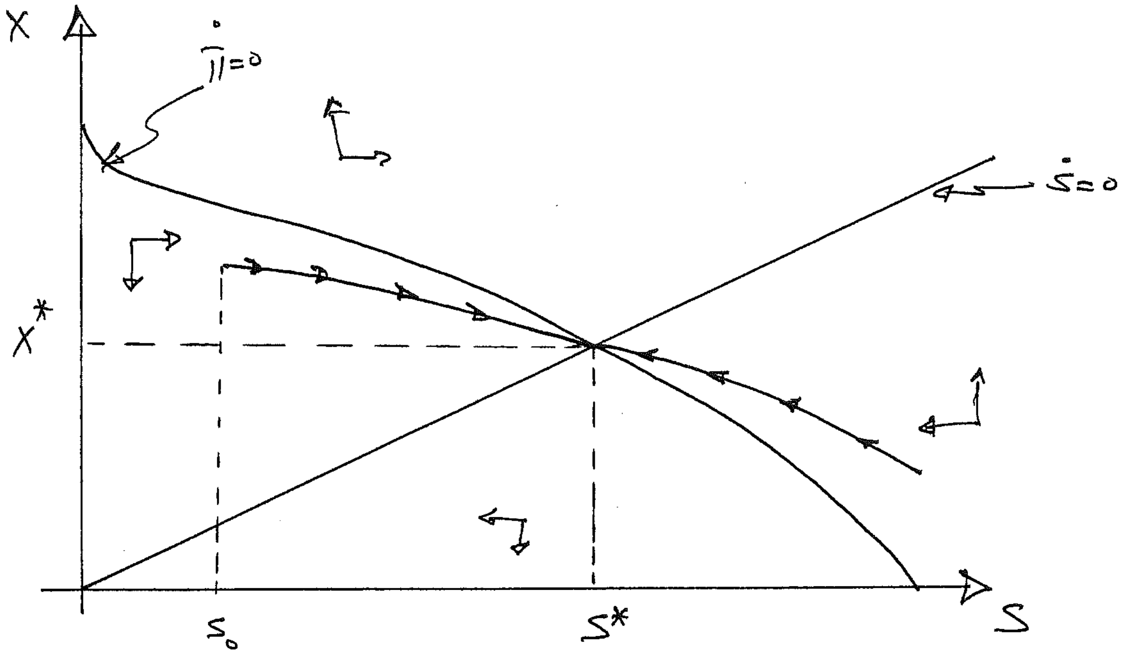

Figure 6: The phase-diagram for the stock pollutant model.

1. Start in quadrant II and draw the difference between marginal benefits and damages, i.e. $B'(x) - D'(x)$.
2. Go to quadrant IV, and using the same procedure as above draw the marginal damage curve for the stock pollutant, i.e. $L'(S)$, and translate $L'(S)$ into present value terms in quadrant III.
3. Pick any emission level x and going (counter-clockwise) find the corresponding stock level S. Repeating this step will give the curve in quadrant I denoted $\dot{\pi} = 0$.

Combine the curves for $\dot{\pi} = 0$ and $\dot{S} = 0$ in figure 6. The intersection of the these two curves identifies the steady-state solution (S^*, x^*).

I have also included the direction of movement in the diagram. Below $\dot{S} = 0$ does S decline, but increases above. Emissions increase to the right of $\dot{\pi} = 0$ and decreases to the left. Thus there are two stable arms in the phase-diagram:

1. $S_0 < S^*$ implies $x(t) > x^*$, i.e. a situation with decreasing emissions, but increasing stocks, or
2. $S_0 > S^*$ implies $x(t) < x^*$, i.e. a situation with increasing emissions, but decreasing stocks.

6 Conclusions

The rule for efficient emissions in the steady-state extends the static model in a natural way; the marginal damage function must be extended with the present value of future damages caused by increased stock of pollutant.

Policy implications:

- the lack of current markets leaves the damages caused by emissions unaccounted for.
- the lack of future markets leaves the damages caused by the stock of pollutant unaccounted for.