Lecture 18: Making non-cooperative games cooperative (1): The Folk theorem

- Objectives
  - show how non-cooperative single shot games can yield cooperative outcomes when they are made dynamic = demonstrate the Folk theorem
  - applicability and limitations of the Folk theorem

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Outline

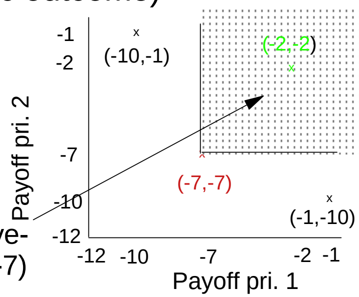
- Repetition - the Nash equilibrium
- Cooperative outcomes in non-cooperative settings (the Folk theorem)
  - mathematical derivation of the Folk theorem
  - graphical presentation
- The Folk theorem in a RAM setting
- Applicability and limitations of the Folk theorem

Nash equilibrium - repetition (1)

- Definition Nash equilibrium: The outcome that results when a player plays his/her **best reply strategy** given that all the other players play their best reply strategy
- Problem: Nash equilibria are rarely Pareto-optimal (in that sense a pessimistic outcome)

	Prisoner 1	
Prisoner 2:	Don't accuse	Accuse
Don't accuse	(-2,-2)	(-1,-10)
Accuse	(-10,-1)	(-7,-7)

Region of potential Pareto improvement from non-coop solution (-7,-7)



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The Folk theorem (1)

- Demonstrates how cooperative outcomes (that differ from the single shot Nash equilibrum) may occur in noncooperative settings
- Requirement: infinitely repated games
  - ... or a game with random stop time [has same effect as infinite stop time as backwards recursion then is not applicable]
- <u>Definition of the Folk theorem</u>
  Any individually rational pay-off vector can be supported as a Nash equilibrium in repeated games that last forever and the discount rate is sufficiently low.

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... the Folk theorem (2)

Dynamic concept - main intuition

$NPV_{nice} \geq NPV_{bad}$ for all agents in the game

nice bad

$$\sum_{t=o}^{\infty} \beta_i^t \pi_{i,t}^c \geq \left(\sum_{t=o}^{T-t} \beta_i^t \pi_{i,t}^c\right) + \beta_i^T \varphi_{i,T} + \left(\sum_{t=T+1}^{\infty} \beta_i^t \pi_{i,t}^n\right) \quad [1]$$

β_i the discount factor, $\frac{1}{1+r_i}$, for agent *i*

$\pi_{i,t}^c$ the payoff to agent *i* of playing cooperatively in period *t*

$\varphi_{i,t}$ the best reply strategy to agent *i* given that the other players play cooperatively in period *t*

$\pi_{i,t}^n$ the payoff to agent *i* when all agents pay non-coop

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... the Folk theorem (3)

Solving [1] is complicated (non-linear). [1] can be divided into a series of 2-period games, and each 2-period game needs to satisfy the

$NPV_{nice} > NPV_{bad}$ criterion

Reducing [1] to a 2-period sub-game:

$$\sum_{t=o}^{1} \beta_i^t \pi_{i,t}^c (= \beta^0 \pi_{i,0}^c + \beta^1 \pi_{i,1}^c) \geq \beta_i^0 \varphi_{i,0} + \beta_i^1 \pi_{i,1}^n = \varphi_{i,0} + \beta_i \pi_{i,1}^n \quad [2]$$

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... the Folk theorem (4)

The solution to [2] in a setting where
$t = 0$ and $t + 1 = 1$:

$$1 > \beta_i \geq \frac{\varphi_{i,0} - \pi^c_{i,0}}{\pi^c_{i,1} - \pi^n_{i,1}} \quad \forall i \in I \qquad [3]$$

The general format, where t can take on any value within the unknown timeframe of the game, T

$$1 > \beta_i \geq \frac{\varphi_{i,t} - \pi^c_{i,t}}{\pi^c_{i,t+1} - \pi^n_{i,t+1}} \quad \forall\ i \in I, \forall\ t \in T \qquad [3']$$

If [3] (or [3']) holds for all agents, it is in all the agents' best self interest to play "nice"

i.e., a cooperative outcome in a non-cooperative setting is achieved

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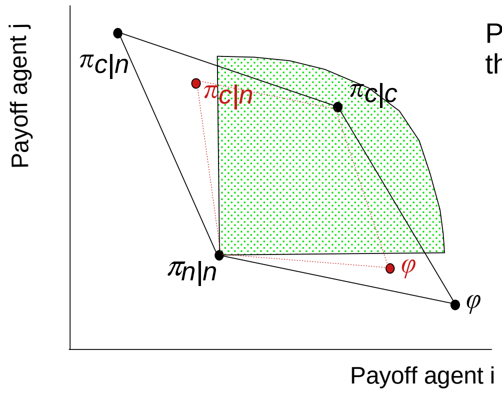
... the Folk theorem (5)

Graphical representation (from agent i's perspective)



Profit ranking for the Folk theorem to make sense:

$\varphi > \pi_{c|c} > \pi_{n|n} > \pi_{c|n}$

The less spread out in NW-SE directions, the more likely it is that the Folk theorem holds (cfr. [3])

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Are the RAM criteria met (1)

1. the participation constraint (individual rationality)
   - yes, as the payoff from participating are not lower than if not participating
2. informational viability
   - yes, if agent *i* can observe the the actions of agent *j* (or other agents) in the following ($t + 1$) time period
   - ... which is an information constraint for the Folk theorem to hold (= agent *i* to respond as required)
3. incentive compatibility
   - yes, if equation [3] (or in general form [3']) holds for all agents [because then it is in all agents self interest to cooperate]

... are the RAM criteria met (2)

The outcome is desirable (over the status-quo):

4. Informationally efficiency
   - yes, as it does not require more information collection and processing than in the initial state
5. Second Best Pareto optimality
   - may not be met, but a clear improvement in welfare for all agents over the status-quo
6. relation to the budget constraint of P
   - there is no principal necessary in the typical Folk theorem setting ⇒ the question is irrellevant
   - ... although the Folk theorem may also be used in game settings where there is a principal present

Applicability & limitations (1)

- Analyze cooperation (or the lack of cooperation) among agents in repeated games
  - examples:
    - market collusion (cartels)
    - teams approaches for reducing nonpoint source pollution from agriculture (a repeated game of cooperation among farmers)
- Limitations
  - the stop time must be unknown (if not, the Folk theorem breaks down due to backwards recursion)
  - sub game perfectness criterion gives a very restrictive outcome for cooperation to take place (as it does not consider future time periods in the sub-game form)

... applicability & limitations (2)

- FT generally thought of for dynamic game settings without a principal (regulator in an env.econ sense) ... but regulators can use FT insights
  - ... to induce compliance and lower monitoring costs (f.ex. facilitating self regulation) in dynamic games :: parallels to reputation based models
  - ... to specify contract terms/game structures that are more likely to meet the RAM criteria
- FT and static games :: can static games be made dynamic, and hence reap some of the benefits of the FT?

Concluding remarks

- Cooperative outcomes can be achieved in repeated games through the Folk theorem
  - random stop time
  - payoff difference between the best reply strategy (Nash setting) and cooperation is not too large
  - the discount rate is not too large
- The Folk theorem is applicable to a special class of repeated games
  - with or without a principal
- Has inspired research to look for other possible cooperative outcomes in other settings

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Concept questions

- Think of some environmental problem that is perceived as static
  - how can this problem be made dynamic?
  - how can the game structure be adjusted to reap the FT benefits?

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