ECN 371: Exercise set 3 - suggested answers

(a) Show that the optimal emission level equals 400

To find the total marginal abatement cost curve, $\sum_i MAC(z_i)$one needs to make a horisontal summation of all the firms' individual marginal abatement cost curves. Consider the two types marginal abatement cost curves (where sub-script *i* is replaced by subscripts *a* and *b*):

- type A: $MAC_A = 10 - z_a$

- type B: $MAC_B = 5 - \frac{1}{2} z_b$

To be able to undertake the horizontal summation (summing over the z_i^*), one needs to introduce the price, $p = MAC_i(z_i)$, and rewrite the two MAC curves as follows

- type A: $p = 10 - z_a \quad => \quad z_a = 10 - p$

- type B: $MAC_B = 5 - \frac{1}{2} z_b \quad => \quad z_b = 10 - 2p$

to obtain inverse demand functions, where type A firms are affected for prices in the whole range $0 \leq p \leq 10$, while type B firms are only affected for prices equal to or below 5, i.e., $0 \leq p \leq 5$. This makes horizontal summation a bit tricky. Think of the MAC curves as the firms' demand of permits. It then follows that for emission permit prices above 5, only type A firms will demand permits (for such high emission permit prices or taxes), type B firms will emit zero). Hence, start with the firms with the highest MAC curves (here type A firms). For the 50 type A firms we get the following aggregate demand:

$$Z_a = 50\, z_a = 50(10 - p) = 500 - 50p \qquad \text{for } 10 \geq p \geq 0 \qquad [1]$$

which also equals the aggregate inverse demand for permit prices $p > 5$.

For type B firms the aggregate demand for permits equals

$$Z_b = 50\, z_b = 50(10 - 2p) = 500 - 100p \qquad \text{for } 5 \geq p \geq 0 \qquad [2]$$

To get the total aggregate demand for $5 \geq p \geq 0$, start by equation [1] and [2] which gives:

$$Z = Z_a + Z_b = (500 - 50p) + (500 - 100p) - 250 = 750 - 150p \text{ for } 5 \geq p \geq 0 \qquad [3]$$

which transforms to

$$p = 5 - \frac{Z}{150} \qquad [4]$$

There is one catch with [4] - it does not account for the fact that typa A firms already demand 250 units. Subtracting the quantity demanded by type A firms when p = 5, i.e., 250 (= 500 - 50(5)) where the two demand curves meet at $p = 5$ then gives:

The total demand (marginal abatement costs) then equals

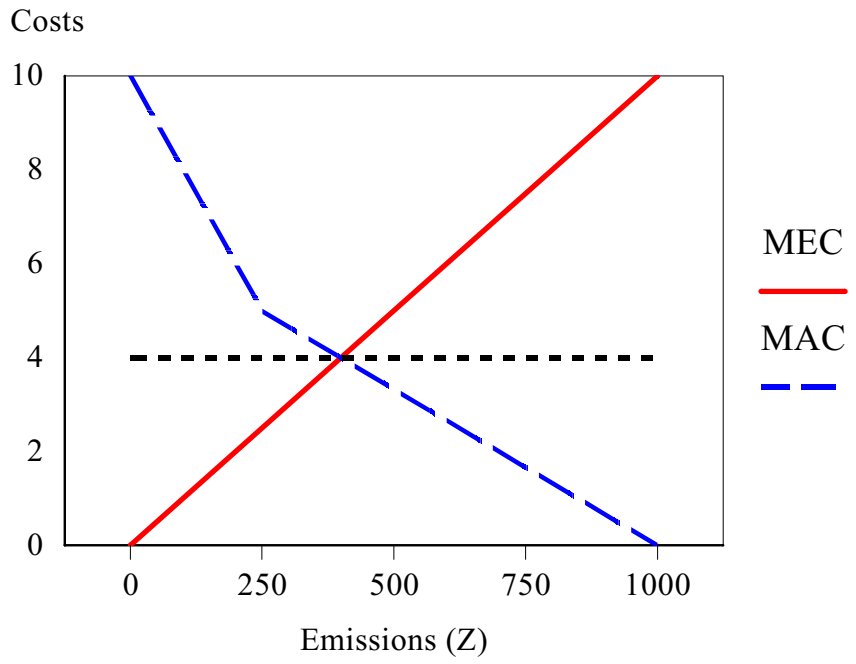
$$0 < p < 5, \text{ i.e., } 750 < Z < 1000 \quad => \quad \sum MAC = p = 5 - \frac{Z - 250}{150}$$

$$5 < p < 10, \text{ i.e., } \quad 0 < Z < 250 \quad => \quad \sum MAC = p = 10 - \frac{Z}{50}$$

Checking for which of the two demand curves in the above equation, leads to

$$MEC = \sum MAC \quad => \quad \frac{Z}{100} = 5 - \frac{Z - 250}{150} \quad => \quad Z^* = 400$$

Graphically, this can be illustrated as follows:



(b) The optimal tax rate

Can be seen from the graph, but can also be solved for. Insert Z^* into the expression for MEC or $\sum MAC$ and solve. Either should give the same answer, i.e., $t^* = 4$.

(c) The optimal emission level for each type of firm.

Set the optimal tax rate, t^*, equal to the type specific MAC curves to get:

type A: $t^* = 4 = 10 - z_a \quad => \quad z_a = 10 - 4 = 6$

type B: $t^* = 5 = 5 - \frac{1}{2} z_b \quad => \quad z_b = 10 - 2 \cdot 4 = 2$

(d) The type of firm who is a sellar or a buyer

Any firm with an optimal emission level less than the initial quota is a seller (in the initial round), and any firm with an optimal emission level larger than the initial quota (in the initial round) is a buyer. Hence, as:

$z_a^* = 6 > 4 = \bar{z}_a \quad =>$ type A firms are buyers

$z_b^* = 2 < 4 = \bar{z}_b \quad =>$ type B firms are sellers

(e) The trade volume and the resulting price, p_Z.

Each firm would my or sell permits until it has reached its optimal emission level. With each firm being awarded $\bar{z}_i = 4$, and all the buyers have the same optimal emission level

($z_a^* = 6$) and all the sellers having the same optimal emission level ($z_b^* = 2$) each firm that buys (or sells) trades to units. With 50 buyers (or sellers), the total amount traded equals 100.

With tradable permits and taxes being equivalent at the margin (and no firm goes bankrupt under the tax regime - there is no indication of exit or entry here), the price in the permit market $p_z = t^* = 4$.

(f) Which of the two systems would the firms prefer?

Under both schemes firms would abate (adjust their emissions to their optimal levels). The marginal abatement costs are the same for both schemes as the integral for total abatement costs are the same for the tax system and the tradable emission permit system.

Type A firms would have to pay taxes amounting to $t^* z_a^* (= 4 \cdot 6 = 24)$, while their expenses for buying permits equals $(z_a^* - \bar{z}) p_z (= (6-4) \cdot 4 = 8)$. For type B firms it is trival, under taxes they pay, under a permit system they get revenues from permit sales. To conclude, both buyers and sellers prefer a tradable permit system over taxes.

Please note that this conclusion holds as long as some of the permits are grandfathered.

(g) Similarities between tradable emission permits and emission taxes

For the same level of abatement, the two systems are equal at the margin. However, as long as some of the initial quota is grand-fathered, tradable permits gives lower total outlays to firms than taxes. Without any tax reimbursement, tradable permits are preferable from a competitiveness perspective (barring the double dividend argument).

(h) The effect of a transaction fee

It does not matter here if such a fee is put on sales, buys or both. In either case, the transaction fee creates a wedge in the market for emission permits, and creates a welfare loss by reducing the traded volume by the same amount. Draw a graph if you doubt this.