ECN 371: Exercise set 1 - suggested answers

1. a. (i) Technically feasible: All points inside or on the boundary of the production possibility set, i.e. A, B, D and E.

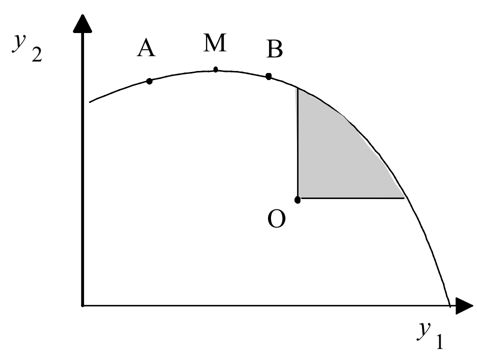
   (ii) Technically efficient: Points D and E (because it is not possible to get more of one of the products without getting less of the other). Note that B is not technically efficient. Why?

   (iii) Economically efficient: Point E (because it is where the price line is tangent to the product transformation curve).

   b. Doubling p_1 gives the following price line: $-\frac{2p_1}{p_2}$, which is twice as steep as the initial price line ($-\frac{p_1}{p_2}$). Consequently, one produces more of y_1 and less of y_2.

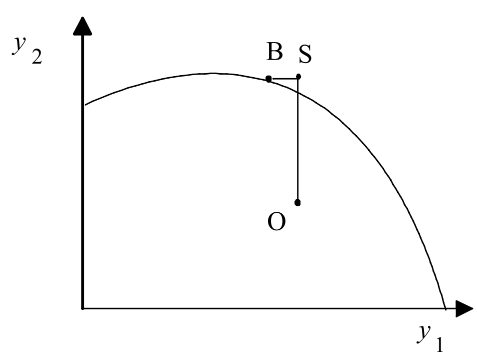
2. a. Allocation O is not efficient because it is possible for both members of the economy to get more without the other getting less.

   b. The region for Pareto improvement is given by the shaded area northeast of O:



   c. We would never observe allocation A given our assumptions because both individuals would increase their utility moving from A to the point M.

   d. It would be possible to move to B from O if we allow for side payments (see below):



   Here, individual 2 gains OS, while individual 1 looses SB. As OS > SB, individual 2 can pay individual 1 the amount SB to make him/her indifferent with his/her initial allocation, and individual 2 would be left with a surplus OS - SB.

e. Outcome B is not a likely outcome for two reasons. First, total wealth is larger in any point located on the segment of the region for Pareto improvements. Second, with the compensation SB, individual 1 is equally well of as he/she was initially. Knowing the large gains to individual 2, it is at least possible for 1 to demand a slightly larger compensation than SB. This would make both individuals better off than they were initially at allocation O.

3. a. The profit function, $\pi = py - (ay + by^2)$

b. Differentiate the profit function by y and set equal to zero (First Order Condition, FOC), then solve for y:

$$\frac{\partial \pi}{\partial y} = p - a - 2by = 0 \quad \Rightarrow \quad -2by = -(p-a) \quad \Rightarrow \quad y = \frac{p-a}{2b}$$

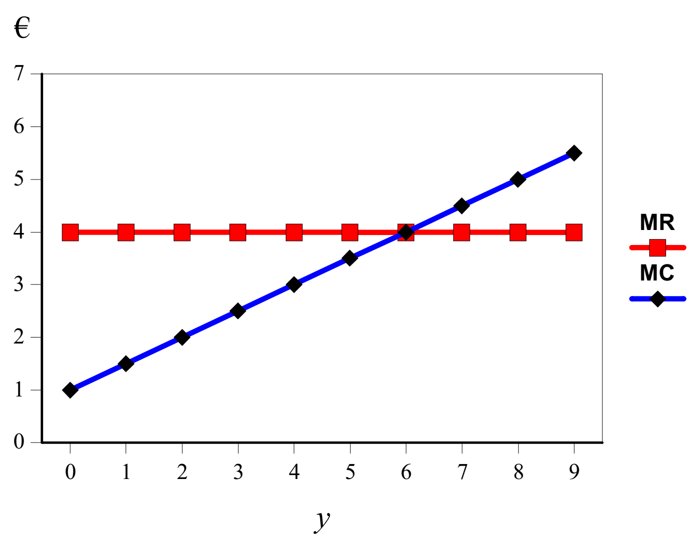
A check of the second order condition (SOC) reveals that it is everywhere negative $\frac{\partial (p - a - 2by)}{\partial y} = -2b < 0$, i.e., we have a global maximum where the FOC holds.

c. Before drawing the graphs, consider the TR and TC functions.

$$TR = py \quad \Rightarrow \quad MR = \frac{\partial TR}{\partial y} = p$$

$$TC = ay + by^2 \quad MC = \frac{\partial TC}{\partial y} = a + 2by$$

which graphed yields



As MC crosses MR from below and SOCs hold everywhere, it appears that the optimal solution is at $y = 6$. Inserting the parameters in the solution to part b gives the same answer: $y = \frac{p-a}{2b} = \frac{4-1}{2 \cdot 0.25} = 6$

The corresponding profits are $\pi = py - (ay + by^2) = 4 \cdot 6 - (1 \cdot 6 + 2 \cdot 0.25 \cdot 6^2) = 9$

d. The profit function with the emission tax

$$\pi = py - (ay + by^2) - tz = py - (ay + by^2) - t\tfrac{1}{2}y^2 = (p - a)y - (b + \tfrac{1}{2}t)y^2$$

e. Solving for the FOCs of the expression in part d:

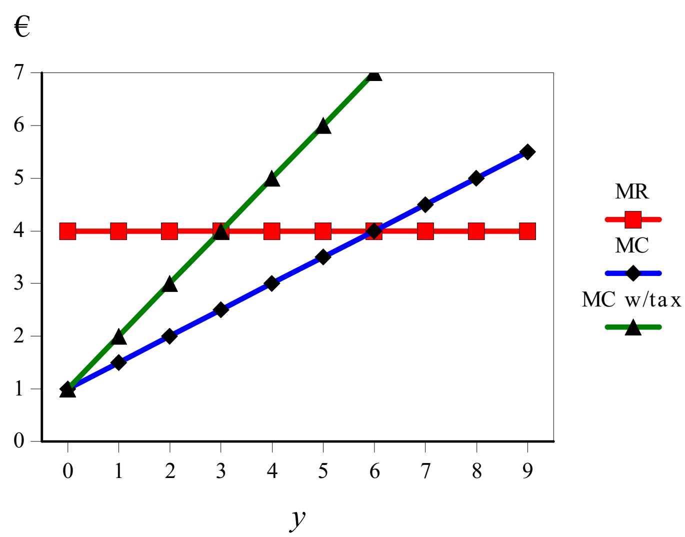
$$\frac{\partial \pi}{\partial y} = p - a - (2b + t)y = 0 \quad \Rightarrow \quad -(2b + t)y = -(p - a) \quad \Rightarrow \quad y = \frac{p - a}{2b + t}$$

f. Inserting parameter values yields: $y = \dfrac{p - a}{2b + t} = \dfrac{4 - 1}{(2 \cdot 0.25) + .5} = 3$

The corresponding profits:
$\pi = py - (ay + by^2) - tz = (p - a)y - (b + \tfrac{1}{2}t)y^2 = (4 - 1)3 - ((2 \cdot 0.25) + 0.5 \cdot 0.5)3^2 = 2.25$

h. Graphical solution with the tax

The TR-function is the same, while the total costs become
$TC = ay + (b + \tfrac{1}{2}t)y^2 \quad MC = \dfrac{\partial TC}{\partial y} = a + (2b + t)y$



The optimal solution with the tax (rotates the MC curve in this problem) is $y = 3$.

i. The maximum allowance on emissions of 12.5 units involves solving the following Lagrangian:

$$L = py - (ay + by^2) + \lambda[12.5 - z] = py - ay - by^2 + \lambda[12.5 - \frac{1}{2}y^2]$$

$$\frac{\partial L}{\partial y} = p - a - 2by - \lambda y = 0$$

$$\frac{\partial L}{\partial \lambda} = 12.5 - \frac{1}{2}y^2 = 0$$

After solving these FOCs simultaneously we get $y = 5$ and
$\lambda = (4 - 1 - 2 \cdot 0.25 \cdot 5)/5 = 0.1$

Inserting the solution for y into the profit function yields :

$$\pi = py - (ay + by^2) = 4 \cdot 5 - (1 + 2 \cdot 0.25 \cdot 5^2) = 6.5$$

j. When the emission cap (maximum allowance is expanded to 24), it suffices to see that the Lagrangian multiplier constraint

$$\frac{\partial L}{\partial \lambda} = 24 - \tfrac{1}{2}y^2 \leq 0$$

becomes non-binding vis-a-vis the solution in part c as $y^2 \geq 48$ implies that y is larger than 6, the unconstrained solution. Hence, the constraint is non-binding and the value is the same as for the unconstrained solution. (Those who doubt the result, work out the problem using Kuhn-Tucker).