

## Suggested answer

This brief summary on how to approach the problem only deals with the trade part of the question, not the issue of investment.

The key issue for each firm is to figure out what their marginal abatement costs are for various emission levels, and prior to prices being posted on the exchange, if they should post an offer to sell or buy permits.

A single firm's marginal abatement costs with the 50% reductions in emissions implies that  $z_i = \frac{1}{2} z_i^o$ . The marginal abatement costs for a single firm before any permit trading occurs is then given by:

$$MAC_i(z_i) = \frac{z_i^o - z_i}{\sqrt{z_i^o}} = \frac{z_i^o - \frac{1}{2}z_i^o}{\sqrt{z_i^o}} = \frac{\frac{1}{2}z_i^o}{\sqrt{z_i^o}} \quad [1]$$

To figure out if one is to post an offer to sell or a bid to buy permits, the firm owner calculates the mean *MAC* for the rest of the market by the following formula:

$$MAC(z_{rest}) = \frac{\frac{1}{N-1}((Z^o - z_i^o) - \frac{1}{2}(Z^o - z_i^o))}{\sqrt{\frac{z^o - z_i^o}{N-1}}} = \frac{\frac{1}{2(N-1)}(Z^o - z_i^o)}{\sqrt{\frac{z^o - z_i^o}{N-1}}} \quad [2]$$

where  $Z^o$  denotes aggregate emissions (= 434),  $N$  denotes total number of firms (= 5), and  $z_i^o$  denotes firm *i*'s emissions. Using formulae [1] and [2] gives the following table:

Firm <i>i</i> 's initial emission level ( $z_i^o$ )	Firm <i>i</i> 's pre-trade MAC (from [1])	Rest of firms mean pre-trade MAC (from [2])	Advice to firm <i>i</i> before trading starts
64	4,00	4,81	offer sell one unit, $p > 4.81$
81	4,50	4,70	offer sell one unit, $p > 4.70$
100	5,00	4,57	bid buy one unit $p < 4.57$
121	5,50	4,42	bid buy one unit $p < 4.42$
144	6,00	4,26	bid buy one unit $p < 4.26$

Note that firms in the initial round should not offer to sell or buy at their pre-trade MACs, but rather look at the mean price in the rest of the market, and that one initially should not sell or buy more than one unit (just to get trades going and learn about the market). Also note that as each firm calculates the mean price in the market given its own emission level, it should expect that the other firms' MACs are distributed around that mean. To try to extract some rents, one should therefore offer to sell (if one is a potential seller) at a higher rate than this mean, and to buy (if one is a potential buyer) at a lower rate than this mean.

In the given exercise, the distribution of initial emission levels where 1 firm w/ 144 units, 2 firms with 81 units and 2 firms with 64 units. Consequently, all but the large firm should offer to sell.

A little trick question: Before investments in the new technology is made, why is the expected equilibrium price in the market (i.e., the price when all rents from trade have been absorbed) somewhere between 4.57 and 4.70?

Hint: look at the table on the previous page.