Game Theory and Resource Allocation Mechanisms[1]

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1 Introduction

Asymmetric information is a key issue in modern regulation theory. Specifically, the regulator (the principal, $\mathcal{P}$, in the game theory terminology) is less informed about firms or individuals (the agents, $\mathcal{A}$, in game theory terminology). If the regulator had full information about the firms or individual whose behavior the regulator wants to change, designing regulations would be a lot more simple. Fortunately, the regulator does not have full information about our preferences and actions. Those in doubt on this should read George Orvell's book *1984*, which deals with a society where "big brother" sees almost everything persons do.

This note proceeds as follows. First, it looks at the familiar Nash equilibrium, which gives quite grim perspectives for cooperation between agents, or the principal's possibilities of influencing agent behavior. Next, it looks at the two major types of information asymme-

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[1] For a more detailed introduction to game theory, confer with any textbook in game theory or Rasmusen (1989). An excellent current summary of applied game theory is Gibbons (1997).

tries, adverse selection and moral hazard, in the relation between the principal and agents (or between agents). This will be dealt with in the context of principal-agent (P/A) models. This leads to the notion of resource allocation mechanisms (RAMs), a more modern variation of P/A models. Finally, the note deals with how one can model cooperation between agents.

2 The Nash equilibrium – a brief repetition

The Nash equilibrium deals with situations where each agent in the game plays his/her best reply strategy given that the other players in the game also play their best reply strategy. Intuitively, it builds on the notion that when the actions of one agent influences the payoff (utility, welfare) of other agents, it is almost self evident that sophisticated agents will consider the actions of these other agents when making their own decisions. The classical example in this case is the Prisoners' Dilemma.

The Prisoners' Dilemma consists of two prisoners (suspects) who are suspected of having committed an armed robbery[2]. Moreover, the State attorney has sufficient evidence to have both suspects sentenced to one year in prison, but insufficient evidence to get the two suspects found guilty of the more serious crime that took place. In the US the penalty can be reduced if one suspect cooperates with the State attorney, i.e. witnesses against the other suspect(s). Assume that in this case this results in 0 years (freedom) for the one that cooperates (accuses the other suspect), and 10 years in prison for the one who does not cooperate. If both suspects cooperate, assume that they both get 7 years of prison. The two suspects cannot communicate with each other (coordinate their strategies). Table 1 shows the actions and payoffs for the two suspects.

Table 1: The prisoners' dilemma

		Suspect no. 1	
		Don't accuse suspect 2 (cooperate)	Accuse suspect 2 (don't cooperate)
Suspect no. 2	Don't accuse suspect 1 (cooperate)	(-2,-2)	(-1,-10)
	Accuse suspect 1 (don't cooperate)	(-10.-1)	(-7,-7)

[2] The setting is in the US, where the state attorney can reduce or even let off suspects who cooperate by serving as a witness for the state in getting other suspects found guilty.

It is almost self evident that the most desirable outcome for the two suspects is that neither accuses the other, and they both only get one year in prison (payoff equals -1). However, this is a highly unlikely outcome in a single shot game with the payoffs depicted in Table 1.

Ignorant of the actions of suspect 2, suspect 1 has to decide what to do. The rational approach for suspect 1 is to consider both actions of suspect 2. Assuming that suspect 2 has not cooperated with the state attorney, the best action for suspect 1 is to accuse suspect 2 (cooperate with the state attorney), because the payoff for suspect 1 from the action "accuse" (= 0 years in prison) exceeds the payoff from not accusing suspect 2 (= 1 year in prison). Similarly, assuming that suspect 2 has accused suspect 1, the best action for suspect 1 is to accuse suspect 2 (7 years in prison is not as bad as 10 years in prison). In brief, the action "accuse suspect 2" dominates the action "don't accuse suspect 2". Viewing this game from suspect 2's perspective yields the same outcome, i.e., that "accuse the other suspect" is a dominant action. Therefore, the action ("accuse suspect 2", "accuse suspect 1") is a dominant strategy equilibrium.[3] In this particular case this also corresponds to the Nash-equilibrium. In the prisoners' dilemma equilibrium it is noteworthy that if the two suspects could commit to the "don't accuse the other suspect", they would both achieve a higher payoff (the lower prison sentence of one year each against the Nash payoff of 7 years each). This is an almost general result in game theory – Nash equilibria are rarely Pareto-optimal.

A Nash equilibrium occurs when all agents play their best reply strategy given that the other agents also play their best reply strategy. For such a reply strategy to be a Nash-equilibrium the following assumptions and rules must be met:

<u>Assumptions</u>:

A1: $s_i \subset R^m$ (strategies are in an m-dimensional strategy space) that is compact and convex (closed and bounded), $\forall i \subset I$ (for all agents in the game)

A2: The payoff, $\pi_i(S) \subset R$, is defined, continuous and bounded $\ldots s_i \subset S$ (for all strategies in the strategy space, S), $\ldots i \subset I$ (for all agents in the game)

A3: The payoff for agent i excluding the optimal strategy, $\circ_i(S \backslash s_i^{\&})$ is concave with respect to $s_i^{\&} \subset S$ (any optimal strategy in the strategy space), $\ldots i \subset I$ (for all agents in the game)

[3] It is important to note that the outcome of the came could be radically changed if the game was played repeatedly (this case is discussed in section 5 "Inducing cooperative behavior in a non-cooperative world").

Rules:

R_1: players cannot make binding agreements (contracts)

R_2: the strategy choice of each player is determined without knowledge of the strategy choice of the other player(s)

Rule R_1 indicates that the concept of a Nash-equilibrium belongs to the class of non-cooperative games (see below). Even if players cannot make binding and sanctionable agreements, cooperative outcomes may exist under certain conditions.

In game theory one distinguishes between two main types of games, (i) cooperative games and (ii) non-cooperative games. The key elements in cooperative games are the possibilities of the players to make agreements, and to sanction the players who breach the agreement. Agreements can be formal (contracts) or informal (no contract). The possibilities of sanctioning any player who does not keep his/her part of the agreement is essential. This is closely linked to the informational structure in the game, i.e. the possibilities to detect which player(s) that breach the agreement. Games that does not belong in the "family" of cooperative games are non-cooperative.

The distinction between non-cooperative and cooperative games is somewhat artificial, because the negotiation process preceding any cooperative game can be modeled as a non-cooperative game (Harsanyi and Selten, 1988; Rasmusen, 1989).[4]

3 Asymmetric information and principal-agent models[5]

In principal-agent models the principal has a coarser information set (less accurate information) regarding the agents' type(s) or action(s) than the agent. There are two types of principal-agent models:

(1) adverse selection models (hard to observe agent characteristics/hidden type), where the principal does not fully know the type of the agent, and

(2) moral hazard models (hard to observe agent actions/hidden action), where the principal does not fully know the action of the agent.

[4] John Nash, John Harsanyi and Reinhard Selten shared the Nobel price in economics in 1995, all for work done on equilibrium criteria in game theory.

[5] For a more complete discussion of principal-agent models, see any recent textbook in game theory like Rasmusen (1989).

The typical textbook example used for adverse selection is the problem an insurance company (the principal) has of knowing if you (the agent) are a good or a bad driver. For the insurance company this information is vital, because if you are a bad driver, the risks may be higher that the insurance company has to pay for damages caused by your lack of driving skills. With the same type of insurance deal for good and bad drivers, good drivers end up subsidizing the bad drivers, which may cause some of the good drivers to choose not to have insurance (except for the minimum required by law - personal damage and objective responsibility on persons/vehicles). Possible solutions for the insurance company in this case include:

(a) Bonus (reduced price for insurance) for drivers without accidents for a prolonged period of time.

(b) Two types of insurance contracts:
   - type 1 - high payment, low deductible
   - type 2 - low payment, high deductible

Of particular interest in this case is option (b). Provided you know you are a good driver, you will – all other things equal – choose the "type 2" insurance contract. Conversely, if you are a bad driver, you will choose the "type 1" insurance contract. Hence, by having this "menu" of contracts, you have voluntarily chosen to reveal your type to the insurance company. This outcome is called a separating equilibrium.[6] Under certain conditions, the existence of separating equilibria has several desirable features. The most important one being that it increases overall welfare.

Driver behavior is the equivalent example of moral hazard in the auto insurance business. Here, the most common policy is to award bonuses to drives with a long history of driving without accidents (a).

In many cases there is no sharp distinction between adverse selection and moral hazard models. Again, consider auto-insurance. For an insurance company, a good customer is any driver who has a long history of no accidents, i.e. no need for the insurance company to make any damage payments.

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[6] One problem with this type of menus emerge when multiple (> 1) insurance companies compete for the good customers. In this case Rotchild and Stiglitz (1976) showed that there exists no separating equilibrium.

4 Resource Allocation Mechanisms

Any economic system or mechanism is a communication process, where messages are exchanged between agents. Each agent transmits messages to which other agents respond according to their self-interest. A successful resource allocation mechanism (RAM) utilizes this, so that each agent without necessarily understanding the complete process, is induced to cooperate in the determination of a satisfactory bundle of goods and services (Campbell, 1987). As such RAMs are extensions of the principal-agent model. Necessary features of any RAM are: (i) the participation constraint (individual rationality is the term used in the mechanism design literature), (ii) informational viability, and (iii) incentive compatibility. In addition it is desirable that a RAM is able to rank welfare between polices. The main desirable features are (iv) informational efficiency, i.e., that the mechanism processes the necessary information in a least cost fashion, (v) Pareto optimality (or as we shall see, Second Best Pareto Optimality (SBPO) as Pareto optimality is not achievable for incentive compatible mechanisms), and (vi) that it does not exceed the regulator's budget constraint.

The Economy

An economy consists of firms, consumers and government. Firms produce private goods.[7] Government orders and pays for public goods.[8] A sector of an economy concerns the production and consumption of a particular type of goods or services. Consumers seek to maximize utility from consumption of both private and public goods.

Any RAM must be viewed in conjunction with the environment[9], which consists of technology, preferences and institutions. The technology describes the firms' production processes. The resulting constrained profit functions are derived using McFadden's (1978) approach. Moreover it is assumed that the firms behave according to the putty-clay framework (Johansen, 1972). Preferences influence consumers' choices. The basis for the welfare analysis is the indirect utility function, $V_i(p, X_i) \;\; \forall i \in I$ (Varian, 1984). Second-Best Pareto Optimality (SBPO) is chosen as the welfare indicator.[10] The reason for this choice is that designing

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[7] Whenever the term "private goods" is used, it refers to goods and services that are rival and exclusive in consumption (Randall, 1983).

[8] Whenever the term "public goods" is used, it refers to non-rival and non-exclusive goods and services in consumption (Randall, 1983).

[9] The term "environment" refers to the economic environment.

[10] Several other welfare indicators exist. These include Aggregate Money Metric Utility, the Bergson-Samuelson Social Welfare Function, and Pareto-Optimality.

incentive compatible regulations has its cost, making Pareto optimality non-attainable. SBPO can be defined to only include the set of incentive compatible regulations (mechanisms). Institutions include the legal system and the organization of government.

Criteria for Resource Allocation Mechanisms

The importance of these properties, and necessary modifications due to conflicts between them will be demonstrated.

The *participation constraint* (individual rationality) requires that the suggested RAM generates allocations that make all the firms, $n \in N$, and all consumers, $i \in I$, at least as well off as they would be in the reference scenario. Care must taken in terms of defining the reference scenario. Often the reference scenario is specified as "given that a regulation is needed". In this case, the participation constraint implies it pays for agents to take part in the game given that the regulation is in place. One example of this is trading in a tradable permit scheme given that the market has been established. Such a definition may, however, miss the point that all environmental regulations imply some costs to some agents. Consequently, these agents (and in particular firms) are likely to combat the introduction of the regulation, implying delayed implementation.

Informational viability is important because RAMs that do not satisfy this property have informational requirements that exceed the available information. Any RAM that is not informationally viable may therefore not yield its intended outcome(s). Informational viability requires (i) that agents only use accessible information about the other agents, and (ii) that the amount of information is such that it can be treated (Campbell, 1987). Formally (i) is called the privacy preserving property of the RAM, implying that only public information about one agent can be used by the other agents. A convenient way of formalizing (ii) is that the message space of the proposed RAM must be a finite Euclidian space. This means that the vector of information exchanged between the agents has finite dimensionality.

Incentive compatibility means that it should be in the self interest of the firms to act in the prescribed way. Unfortunately joint incentive compatibility and Pareto-optimality are not always possible. The following theorem due to Hurwicz (1972) illustrates this (Campbell, 1987, p. 114):[11]

<u>THEOREM 1</u>: Let R' be a mechanism defined on a family of economic environments, the family of self-regarding utility functions that exhibit diminishing marginal rates of substitution everywhere. If for every environment within this family of environments, the mechanism R' generates equilibrium allocations that are Pareto optimal and individually rational (participation constraint met), then it can be manipulated.

<u>DEFINITION 1</u>: Manipulation means that an agent by not revealing his true preferences is able to increase his own welfare, resulting in a loss of welfare for other agents.

The proof of Theorem 1 assumes that one agent can obtain increased profits (or utility) by manipulative behavior when all the other agents behaves sincerely. Thus, each agent is led to behave manipulatively, and the outcome is not Pareto optimal. Consequently, if the policy maker chooses a RAM that could lead to a Pareto optimal outcome, such an outcome is not guaranteed. Now suppose that the policy maker decides to opt for an incentive compatible RAM. Even if Pareto-optimality may not result, the proposed RAM will yield a predictable outcome, whose welfare properties can be evaluated.

According to the theory of second best, it is uncertain whether applying marginal cost pricing in the sectors under consideration will move the entire economy closer to the Pareto optimum, unless the optimum conditions are met in the rest of the economy (Lipsey and Lancaster, 1956; Boadway and Bruce, 1984). In general, the latter will not be the case. Thus, Pareto-optimality may not be applicable in the case of RAMs seeking to correct for externalities. Spulber (1989) suggests replacing Pareto-optimality with Second-Best Pareto-Optimality (SBPO). Expressed in terms of the constrained indirect utility function, SBPO is defined as:[12]

<u>DEFINITION 2</u>: Second-Best Pareto-Optimality: An externality vector $\mathbf{Z}_a$ is SBPO if there exists no other externality vector $\mathbf{Z}_b$ such that $V_i(\mathbf{p}_b, X_{ib}|\mathbf{Z}_b) \geq V_i(\mathbf{p}_a, X_{ia}|\mathbf{Z}_a)$ $\forall i \in I$ and $\exists\ i \in I$ such that $V_i(\mathbf{p}_b, X_{ib}|\mathbf{Z}_b) > V_i(\mathbf{p}_a, X_{ia}|\mathbf{Z}_a)$, where $\mathbf{p}_a$ and $\mathbf{p}_b$ denote the

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[11] The proof of this theorem can be found in Campbell (1987, pp. 114-115).
[12] A definition of SBPO using the ordinary utility function is found in Spulber (1989, p. 355).

price vectors from the externality vectors $\mathbf{Z}_a$ and $\mathbf{Z}_b$ respectively, and X_{ia} and X_{ib} denote the associated consumer incomes for the ith consumer.

The difference between Pareto optimality and SBPO is that while the former denotes a situation on the grand utility frontier, the latter is not quite on the frontier. In stead it focuses on the existence of other feasible allocations. SBPO occurs when there exists no other feasible allocation which gives a higher utility (Spulber, 1989).

Informational viability and efficiency and incentive compatibility are required for the proposed RAM to yield a predictable outcome. The participation constraint is important to facilitate the implementation of the RAM. To evaluate any RAM, a welfare indicator is needed. SBPO is chosen as the welfare indicator because it does not require the RAM to correct for all inefficiencies in the economy, and it does not require individual utilities to be comparable.

With SBPO chosen as the welfare indicator, it remains to be demonstrated that informational efficiency is a necessary condition for SBPO (and hence a desirable property for the RAM as a whole).

<u>PROPOSITION 1</u>: Informational efficiency is a necessary criterion of SBPO.

<u>PROOF</u>: Suppose informational efficiency is not necessary for SBPO. Let $C^I(..)$ denote the informational costs. Let there exist an externality vector, **Z**', which is SBPO, while the proposed RAM, R' is not informationally efficient. Also assume there exists another RAM, R" which results in the same externality vector **Z**' at less informational cost, i.e. C^I(R") < C^I(R'). The difference in informational costs between the RAMs is then $\Delta\, C^I$(R',R") = C^I(R') -C^I(R") > 0, which can be used to make some or all the agents better off. Then by the definition of SBPO, the RAM R' is not SBPO. Q.E.D.

Informational efficiency means that there exists no known RAM which satisfies the stated objectives at less cost of gathering and processing (Campbell, 1987). Once a usable definition for efficiency of the proposed RAM has been obtained, it will be shown that informational efficiency is a necessary criterion for efficiency of the proposed RAM.

The necessary modified properties of a RAM (for yielding a predictable outcome) are therefore:

(i) participation constraint met (individual rationality),

(ii) informational viability, and

(iii) incentive compatibility, and

Desirable criteria in terms of social welfare are:

(iv) informational efficiency,

(v) second-best Pareto-optimality (welfare indicator), and

(vi) within the principal's budget constraint

5 Inducing cooperative behavior in a non-cooperative world

To this point this note has dealt with non-cooperative outcomes in game theory, and the RAM extension of the P/A model for obtaining predictable outcomes. This section focuses on criteria for getting cooperative behavior in non-cooperative games. There are two ways this may take place: (i) provided some conditions on the payoffs and the nature of the game are met, repeated games can result in cooperative behavior, or (ii) with the use of side payments games without a clear repetitive nature may yield cooperative outcomes. Already at this stage it should be noted that cooperative outcomes in repeated games are easier to achieve than in non-cooperative games.

5.1 Cooperative behavior in repeated games

In real life as well as in theory it is possible to show that cooperative outcomes (outcomes that differ from the single shot best reply strategies associated with the Nash equilibrium) may occur in non-cooperative settings. This is known as the Folk theorem (Rasmusen, 1989):

Any individually rational payoff vector can be supported as a Nash equilibrium in repeated games that last forever and the discount rate is sufficiently low.

In economic terms this means that if the net present value of playing cooperatively (NPVc, the left-hand side of [1]) exceeds the net present value of playing basic non-cooperatively

(NPVa, the right-hand side of [1]), players will choose to play cooperatively. Mathematically this can be expressed the following way:

$$\Sigma_{t=o}^{\infty} \ \beta_i^t \ \pi_{i,t}^c \geq (\Sigma_{t=o}^{T-t} \ \beta_i^t \ \pi_{i,t}^c) + \beta^T \varphi_{i,T} + (\Sigma_{t=T+1}^{\infty} \pi_{i,t}^n) \qquad [1]$$

where β_i is the discount factor $\frac{1}{1+r_i}$ for agent *i*,

$\pi_{i,t}^c$ is the payoff to agent *i* of playing cooperatively in period *t*,

$\varphi_{i,t}$ is the payoff to agent *i* of playing his best reply strategy in period *t* given that the other players still play cooperatively, and

$\pi_{i,t}^n$ is the payoff to agent *i* from playing his best reply strategy given that the other players also play their best reply strategy (Nash).

After reducing [1] to a two-period game (sub-game perfectness), leaving out the first *T-1* time periods where both agents play cooperatively one gets:

$$\Sigma_{t=o}^{1} \ \beta_i^t \ \pi_{i,t}^c (= \beta^0 \pi_{i,0}^c + \beta^1 \pi_{i,1}^c) \geq \beta_i^0 \ \varphi_{i,0} + \beta_i^1 \ \pi_{i,1}^n = \ \varphi_{i,0} + \beta_i \ \pi_{i,1}^n \qquad [2]$$

as $\beta_i^0 = 1$. This can be rearranged the following way:

$$1 > \beta_i \geq \frac{\varphi_{i,0} - \pi_{i,0}^c}{\pi_{i,1}^c - \pi_{i,1}^n} \quad \forall i \in I \qquad [3]$$

In general form in terms of time, *t*, [3] becomes:

$$1 > \beta_i \geq \frac{\varphi_{i,t} - \pi_{i,t}^c}{\pi_{i,t+1}^c - \pi_{i,t+1}^n} \quad \forall i \in I, \forall t \in T \qquad [3']$$

Equation [3] (or [3'] in the general time form) is the condition for the Folk theorem. For [3'] to be applicable the following must hold:

(a) The ranking of the payoffs is $\varphi_{i,t} > \pi_{i,t}^c$ and $\pi_{i,t+1}^c > \pi_{i,t+1}^n$.

(b) The game is repeated, either in infinity or with a random stop time. If this condition is not met, i.e., the stop time is known with certainty, it follows from backwards recursion (compare with [2]) it follows the ranking of the payoffs in (a) that agent *i* should always play his best reply strategy (φ_{T-1})given that the others still play cooperatively (π_{T-1}^c) in period *T*-1. The other agent (j) can deduce this behavior, and therefore plays the same strategy (φ_{T-2}) in period *T*-2.

(c) The agents individual discount rate (r_i) is sufficiently low.

The relative magnitudes of the rankings in (a) is also important in another aspect, namely in terms of the stability of the cooperative solution given if [3'] holds. To see this, reconsider [3'], and assume that the difference $\pi^c_{i,t+1} - \pi^n_{i,t+1}$ is far greater than the difference .$\varphi_{i,t} - \pi^c_{i,t}$. This means that the value of the discount factor, β_i, can be smaller (i.e., the interest rate, r_i, can be larger) for [3'] to hold.

In terms of policy implications this is of particular relevance. Consider a situation where there are two policy options. The first with the higher payoff to agent *i*, $\varphi_H(o_i, c_j)$), when agent *i* plays his best reply strategy, o_i, given that agent *j* cooperates, c_j. The difference in the second policy is that the payoff to agent *i* is relatively lower, $\varphi_L(o_i, c_j)$ from non-cooperative behavior of agent *i*. This is shown in Figure 1.

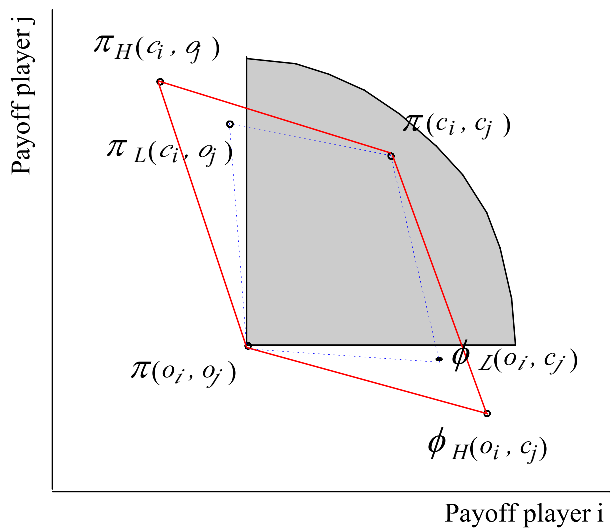

Figure 1: Stability of cooperative outcomes from the Folk theorem.[13]

The more stretched out in the direction southeast - northwest, i.e., the larger the difference between $\varphi_s(o_i, c_j)$ and $\pi_s(c_i, o_j)$ where $s \in \{H, L\}$, relative to the northeast - southwest diagonal of $\pi(c_i, c_j)$ and $\pi(o_i, o_j)$, the less likely it is that the condition holds for cooperation in Folk theorem. To see this, consider [3'] noting the change in notation.

[13] Symbols: possible equilibria = , $\pi(o_i, o_j)$= payoff without cooperation,$\pi(c_i, c_j)$= payoff with cooperation, $\pi_s(c_i, o_j)$ = payoff in state *s* when *i* plays cooperatively and *j* plays best reply strategy given that *i* plays cooperatively,,$\varphi_s(o_i, c_j)$= payoff in state *s* when *i* plays best reply strategy given that *j* plays cooperatively, and s $s \in \{H, L\}$.

It is important to remember that the Folk theorem only is to be applied to situations that can be modeled as repeated games with unknown (random) stop time.

5.2 Cooperative behavior in games without clear repetition

The graphical illustration of the Folk theorem can, however, also be useful in analyzing the likelihood of games that are not repeated. Specifically, this relates to games where the ranking of payoffs resembles the Nash situation. From player *i*'s perspective,

$$\varphi_i(o_i, c_j) > \pi_i(c_i, c_j) > \pi_i(o_i, o_j) > \pi_i(c_i, o_j) \qquad [4]$$

i.e., playing the best reply strategy (non-cooperative when the other player cooperates, $\varphi_i(o_i, c_j)$), gives the highest payoff to player *i*, while playing cooperatively when the other player does not cooperate, $\pi_i(c_i, o_j)$ gives the lowest payoff to player *i*. If this ranking holds for all the players, no player will make the first cooperative move, even if the total payoff is larger from cooperation. The only way to achieve cooperative outcomes in such settings is through the use of side payments or penalties.

There are some limits to these side payments/penalties, namely that no player will play a strategy that makes him worse off than he would be in the status quo non-cooperative setting, i.e., the players *security level*. Figure 2 illustrates the issue in a two player setting with the best reply strategy indicated for player *i*, $\varphi_i(o_i, c_j)$.

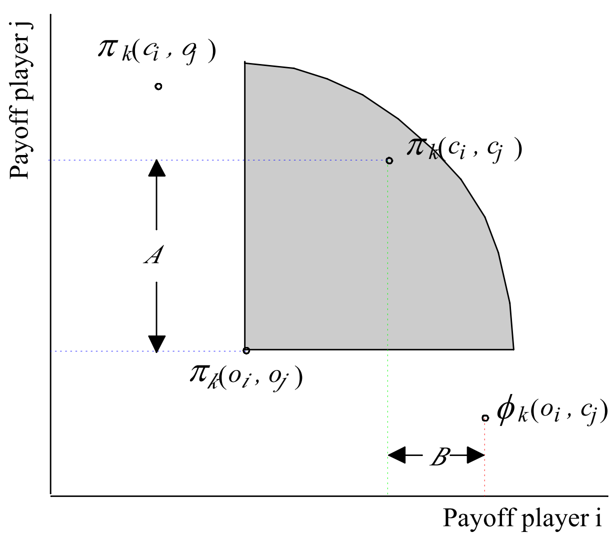

Figure 2: Side payments to achieve cooperative outcomes in non-repeated games.
(*subscript k marks either player i or j*)

Any side payment (or penalty) larger than *B* may induce player *i* to cooperate as this makes the expected payoff from cooperation for player *i*, $\pi_i(c_i, c_j)$, larger than the expected payoff from playing the best reply strategy, $\varphi_i(o_i, c_j)$. Provided that the change in the payoff, *A*, from the non-cooperative outcome, $\pi_j(o_i, o_j)$, to the cooperative outcome, $\pi_j(c_i, c_j)$, of player *j* exceeds B, such a side payment is possible as this does not violate the respective players' *security levels*. However, it is not given that this is in the interest of player *j*. The reason for this is that from player *j*'s perspective the expected payoff from player *i* making the side payment would be much larger.

There are several other ways of reaching the cooperative outcome. One of these is to introduce a set of side payments or penalties that transforms the *Nash type* starting allocation to that of an *assurance game* (Barrett, 1999). In an assurance game, the best reply strategy for all players to other players playing cooperatively, is to cooperate. Figure 3 illustrates this for the two player situation, where the best reply strategy is illustrated by $\varphi_k(c_i, c_j)$. Note that in this case, the southwest - northeast diagonal is longer than the southeast - northwest diagonal in the space connecting the four payoff states. The solution of assurance games are trivial, but reshaping the payoff structure (through side payment or sanctions) is not easy.

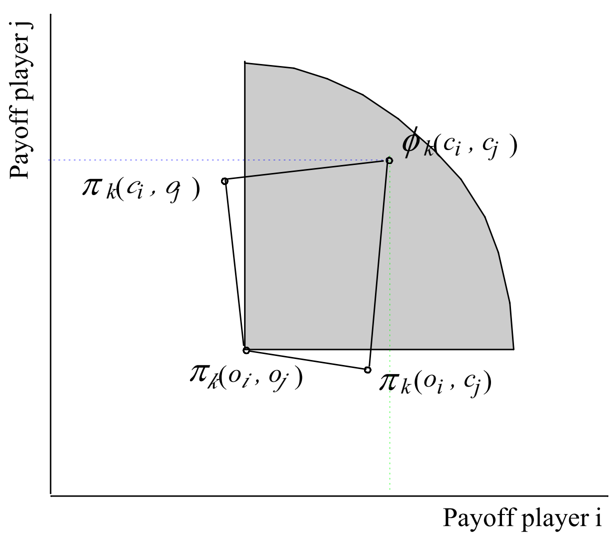

Figure 3: Assurance game (*subscript k marks either player i or j*).

Usually, payoffs are not as symmetrical as the previous graphs has indicated. Consider the following game that is a mix of the assurance game (for player *j*) and a Nash structure from player *i*'s perspective.

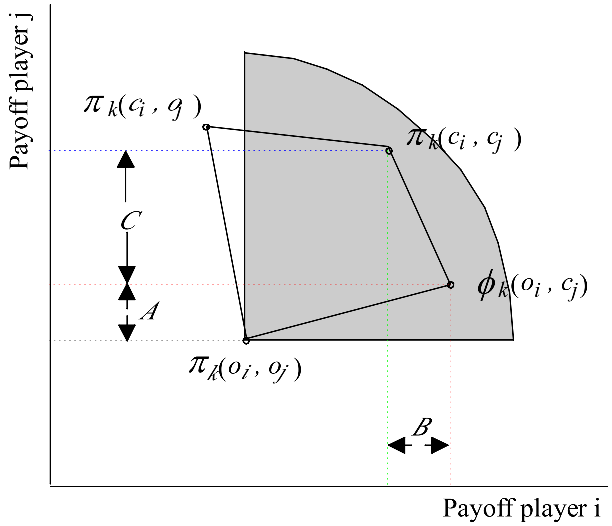

Figure 4: Mixed game with Nash structure for player i, and assurance game for player j.

In this game player *j* plays cooperation (and increases his payoff by *A*). The best response strategy for player *i* is non-cooperation. For a side payment of *B*, player *j* can induce player *i* to cooperate. Provided that the distance *C* is larger than *B*, there is financial room for such a side payment (or a sanction costing the maximum of *C*). Note that even though player *j* may increase his payoff by playing non-cooperatively given that player *j* plays cooperatively, this is an unlikely outcome, as player *i* then reverts to non-cooperation as this costs him nothing compared to his security level.

Generally, any cooperative game or outcome is preceded by a non-cooperative game. The process of achieving to a welfare enhancing equilibrium depends on the relative payoffs of players. The role of side payments or sanctions is primarily to change the game into a type that has a known solution that increases welfare compared to the non-cooperative outcome. There is no standard way of achieving this although the game theory literature provides multiple solution formats that go beyond the scope of this note.

Some of the games we see in regarding environmental issues, for example in terms of international climate negotiations, involve more than two players. One of the critical factors for such game is agreeing on the initial endowment (property rights – in case of the climate issue, the initial quota of climate gas emissions).

Even for such games, that has no principal, the RAM criteria shed extra light on the conditions for achieving equilibria that are welfare enhancing and stable. A key insight relates to the *participation constraint*, i.e., all participants must be better off taking part in the game than staying out. The costs of the policy instruments to be used to create the necessary *incentives* are important in that regard. High cost policies will reduce the net gains, and hence make it less likely that the *participation constraint* is met.

6 Concluding remarks

This note has given a brief overview of game theory. From this note, it should be clear that game theory is not "a cook book" – each case needs to be examined and (hopefully) framed in a setting of a game where the solution is predictable. The types of games that can be solved are rapidly increasing as game theory an area where much research is conducted.

Game theory is a strong tool in terms of making complex issues possible to deal with in a structured fashion. There are two principally different types of games:

(1) *Games with a principal*, i.e., games where the regulator has jurisdiction over agents, but incomplete information on their types and actions. In such games the communication process between the regulator and the agents is critical. The necessary criteria of RAMs - the *participation constraint* (individual rationality in game theory language), *informational viability*, and *incentive compatibility*, provide a starting point for designing policy interments in this setting.

(2) *Games without a principal* are games with no pre-agreed jurisdiction. They can be repeated or have limited repetition. In the repeated case, the Folk theorem is an interesting avenue for investigating the grounds for cooperative and welfare enhancing equilibria. Non-repeated games do not provide this possibility. Hence, non-repeated games involve extra challenges in terms of establishing the endowments (strong parallels to the rights issues in the institutional economics literature), and finding the right mix of side payments and sanctions to give game forms with welfare enhancing stable equilibria.

7 Literature

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