Dynamic Efficiency for Stock Pollutants

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Abstract

With climate gas emissions getting ever higher on the political agenda, clear and sound principles for dealing with stock pollutants are important. This note therefore looks at dynamic cost effectiveness and dynamic efficiency for stock pollutants in environmental policy.

Time consistency and cost effectiveness are in principle easy to reconcile: regulators set emission levels over time so that expected marginal abatement costs for future time periods follow a Hotelling price path. This ensures no arbitrage by shifting abatement from the prescribed targeted emission sequence.

Obtaining dynamic efficiency for stock pollutants is more complicated as in principle two potentially conflicting criteria need to be met: dynamic cost effectiveness, and static optimality in each time period, i.e., that marginal abatement costs evaluated at the chosen aggregate emission level in each time period should equal marginal economic costs at that emission level: $MAC_t(Z_t^*)=MEC_t(Z_t^*)$. Unfortunately, these two conditions may not always jointly be met.

This note argues that one therefore needs to adjust emission levels away from the single period optimal emission levels, Z_t^*, to generate a Hotelling price path. However, discrepancies from the single period optimal emission levels entails some welfare losses. Adjustments therefore need to be made in emission levels where total losses from deviating from the no arbitrage Hotelling price path and the dead weight losses from being off the static optimum in each time period are minimized.

Key words: cost effectiveness, inter temporal cost effectiveness, technological progress, stock effects, cumulative damages, dynamic efficiency, Hotelling's rule, absence of arbitrage.

JEL codes: D90, Q01, Q50.

1 Introduction

The purpose of this note is to clarify what is meant by *dynamic cost effectiveness* and *dynamic efficiency*, and relate these terms to the classical notion of *static optimality*. For starters, *dynamic cost effectiveness* is minimizing the total costs of reaching environmental targets over time, while *dynamic efficiency* is how to correct externalities over time

To get to grips with these terms I will build on concepts that are familiar to those with some background in environmental economics: cost effectiveness, efficiency, and the Hotelling price path (known from the extraction of non-renewable resources). For reasons that will become clear as this paper proceeds, the dynamic price path coinciding with a sequence of static optima no longer holds as demonstrated by Weitzman (2003) and Lin *et al.* (2008) for the mining of resources. The reason for this difference in optima from the standard mining model is the addition of the externality (emissions). This constitutes the main contribution of this paper.

This note is organized as follows. Section two deals with the Hotelling price path. Dynamic cost effectiveness is dealt with in section three, while dynamic efficiency is the topic in section four. Further aspects of the static (section five) and dynamic analyses (section six) follow. Section seven concludes.

2 The Hotelling price path – Hotelling's rule

The Hotelling price path basically describes how a non-renewable resource should be extracted over time to maximize rents from the resource (Hotelling 1931). Imagine that you own a non-renewable resource, like an iron ore, and that your aim is to maximize the rents from this resource. For simplicity, assume that there is no technological progress. Under such conditions your extraction in the various time periods should be such that you could not increase rents by moving some of the sales from one time period to another, i.e., a management regime with the *absence of arbitrage*. This implies that the discounted net price (= the sales price less extraction costs) should be equal between time periods, and is the condition we today denote *Hotelling's rule*.

Let us look a bit more closely at the problem at hand. Suppose that you sold too much in the first time period, so that you get a lower price than what you could have gotten by selling a bit less. At the same time you have reduced future stocks and hence what you can sell in later time periods. This is the topic of Hotelling's (1931) seminal paper. Hotelling's main point is that the net price should have the same time path as you would get from putting the sales income in the bank for a risk free interest rate.

Hotelling (ibid.) uses a mine owner as his demonstration case, and he asks how much iron to mine each year to maximize rents. The iron ore has a known size, which makes it easy to frame the mining problem with the more familiar "cake eating model". Imagine you have a "cake" of the size K that you are to sell over two time periods. In period zero (now) you sell q_0 and in the next time period you sell q_1, such that $q_0 + q_1 = K$ or $q_1 = K - q_0$. Let r be the risk free positive real interest rate. The net present value (NPV) of cake sales then becomes:

$$NPV = p_0 q_0 + \frac{1}{(1+r)} p_1 q_1 = p_0 q_0 + \frac{1}{(1+r)} p_1 (K - q_0) \qquad [1]$$

Assume that prices in the two time periods are the same, i.e., $p_0 = p_1$. Then you should sell all of the "cake" in time period zero because the discounted price in the next time period is lower, and put the sales revenues in the bank and receive the risk free interest rate *r*. This is evident from the following reformulation of [1]:

$$NPV = p_0 q_0 + \frac{1}{(1+r)} p_0 (K - q_0) \qquad [2]$$

where $q_0 = K$ maximizes [2] because $\frac{1}{(1+r)} p_0 < p_0$.

If you are to sell equally much in the two periods, i.e., $q_0 = q_1$, then the prices in the two periods must differ. By differentiating the expression behind the last equation sign in [1] by q_0 (the choice variable) and setting this equal to zero, you get the price path that is required for selling equally much cake in the two time periods and the condition for this sales strategy to be the most profitable:

$$\frac{\partial NPV}{\partial q_0} = NPV'(q_0) = p_0 - \frac{1}{(1+r)} p_1 = 0 \qquad [3]$$

which gives the price path $(1+r) p_0 = p_1$, i.e., the price in time period one must equal the capitalized price in time period zero. This result can be generalized to *t* time periods:

$$p_t = (1+r)^t p_0 \quad \text{or} \quad p_{t+1} = (1+r) p_t \qquad [4]$$

which defines the *Hotelling price path*.

Hotelling's rule in the original version of is written as $r = \frac{\Delta p}{p_t}$. This result emerges by transforming the right hand side of [4] in the following way:

$$p_{t+1} = (1+r) p_t \Rightarrow p_{t+1} = p_t + r\, p_t \Rightarrow p_{t+1} - p_t = r p_t = \Delta p \Rightarrow r = \frac{\Delta p}{p_t} \qquad [5]$$

You may choose to express Hotelling's rule by [4] or [5], but a benefit of [4] is that it gives a price path that clearly is exponential growth as shown in Figure 1.

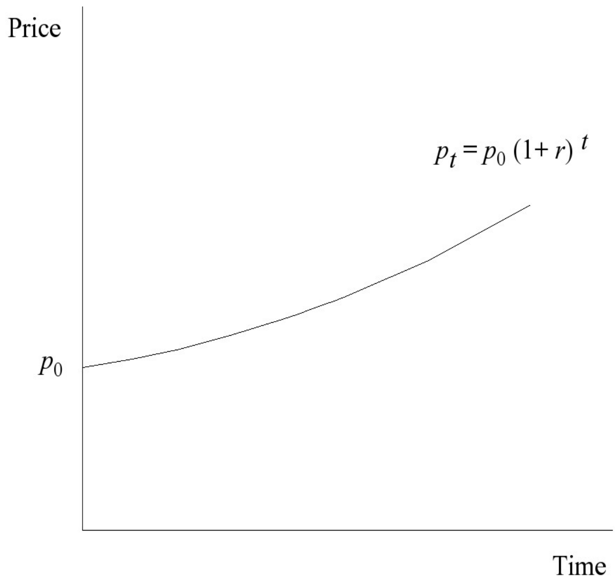

Figure 1: Exponential growth – the Hotelling price path.

Prices – or in our case rents – that follow a Hotelling price path make agents indifferent when to extract the resource as there are no possibilities of increasing rents by shifting extraction from one time period to other time periods. With technological progress, i.e., mining costs decrease over time, profits will be larger in future time periods than today. This implies it will be optimal to shift some of the mining into the future.

3 Dynamic cost effectiveness

Recall the definition of (static) cost effectiveness of reducing emissions: marginal abatement costs evaluated at the chosen emission level should be equal for all agents:

$$MAC_j(z_j^*)=MAC_k(z_k^*), \sum_{i\in I} z_i^*=Z^{cap}\forall i,j,k\in I \quad [6]$$

where z_j^* and z_k^* denote the optimal emission level chosen by agents *i* and *j*, *I* denotes the set of agents in the economy, and Z^{cap} denotes the overall emissions constraint in the economy. As was the case for the Hotelling price path [4] or [5], the above equation is also characterized by the absence of arbitrage, though this time between agents: it is not possible to lower overall costs of reducing emissions by shifting some emissions reductions from one agent to another.

Coupling [6] and the Hotelling price path [4] give the condition for dynamic cost effectiveness:

$$MAC_{j,t}(z_{j,t}^*)=MAC_{k,t}(z_{k,t}^*), \sum_{i\in I} z_{i,t}^*=Z_t^{cap}\forall i\in I,\forall t\in T \quad [7a]$$

$$MAC_{i,t+1}(z_{i,t+1}^*)=(1+r)MAC_{i,t}(z_{i,t}^*)\forall i\in I,\forall t\in T \quad [7b]$$

where $z_{j,t}^*$ and $z_{k,t}^*$ denote the optimal emission level chosen by agents *k* and *j* at time period *t*, $z_{j,t+1}^*$ and $z_{k,t+1}^*$ denote the optimal emission level chosen by agents *k* and *j* at time period *t*+1, *I* denotes the set of agents in the economy, and Z_t^{cap} and Z_{t+1}^{cap} denotes the overall emissions constraint in the economy in time periods *t* and *t*+1 respectively.

One problem with [7a] and [7b] is that they cannot be jointly met if $Z_t^{cap} = Z_{t+1}^{cap}$ (replacing *t* by *t*+1 in [7a] makes this notation evident) unless emissions reductions become more costly over time. This is contrary to the common assumption that there is technological progress over time, implying that costs decrease. As a matter of fact for [7b] to hold in general,

$$Z_{t+1}^{cap} < Z_t^{cap} \quad [8]$$

that is the overall emission cap in the economy becomes tighter. [8] has some profound implications for environmental policy:

1. Setting a tight (low) initial aggregate emissions targets in the economy makes it unlikely that dynamic cost effectiveness would hold in general.
2. Setting a higher initial emissions quota (lower reductions) in an early phase makes it easier to tighten the aggregate targets over time, and thereby ensuring dynamic cost effectiveness. This has the following additional implications:
    1. A gradual phasing in of targets makes it easier to gain (political) acceptance of environmental policies.
    2. The benefits of making it easier to achieve dynamic cost effectiveness needs to be compared against the dead weight losses of not setting initial aggregate environmental targets at their optimum where marginal economic costs (marginal damages) equal to (aggregate) marginal abatement costs.

This brings us to the issue of dynamic efficiency, i.e., how to "adjust the Hotelling price path" when initial environmental targets are not set optimally in a static sense.

4 Dynamic efficiency

Recall that static efficiency is cost effectiveness plus the condition that marginal economic costs (marginal damages) should equal marginal abatement costs. Symbolically this can be expressed the following way:

$$MAC_j(z_j^*) = MAC_k(z_k^*) = MEC(\sum_{i\in I} z_i^*) = MEC(Z^*) \forall i, j, k \in I \qquad [9]$$

where z_j^* and z_k^* denote the optimal emission level chosen by agents *i* and *j*, *I* denotes the set of agents in the economy, and Z^* denotes the optimal emission level in the economy.

Given what we just have learned from dynamic cost effectiveness and that cost effectiveness is a necessary condition for efficiency, how should we reconcile cost effectiveness in a dynamic sense (a gradual tightening of the environmental target) with the welfare losses of not having an optimal initial target in a static sense? Before I go to a more formal analysis, let us look at some possible situations.

First, assume that a statically optimal emission target has been set, and that this emission target implies an accumulation of emissions. This would imply that the MEC curve would move to the left and rotate counter clockwise over time. Moreover assume that there is no technological progress. Under such conditions, it is possible to achieve dynamic efficiency in all time periods if the derived marginal abatement costs in the economy grow according to the Hotelling price path, i.e.:

$$MAC_{t+1}(Z_{t+1}^*) = MEC_{t+1}(Z_{t+1}^*) = (1+r) MAC_t(Z_t^*) = (1+r) MEC_t(Z_t^*) \forall t \in T \qquad [10]$$

Figure 2 illustrates this situation:

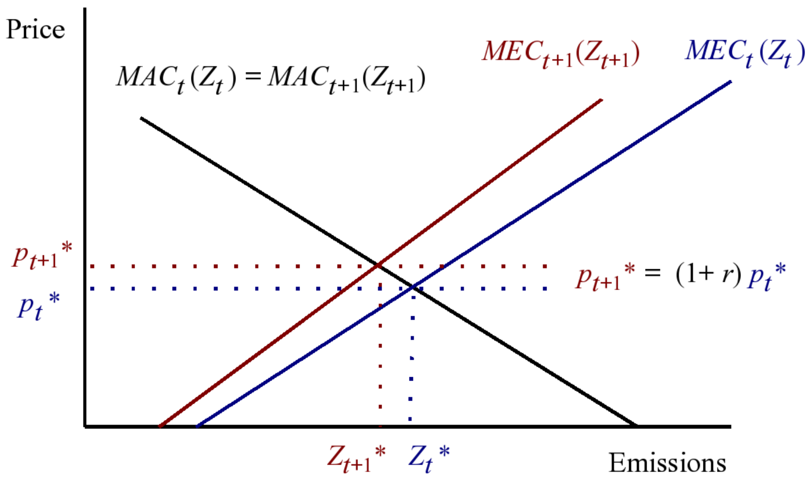

Figure 2: Joint achievement of static and dynamic efficiency.

A striking feature of Figure 2 is that it reproduces our result on how the aggregate emission target declines over time on dynamic cost effectiveness in Section 3, i.e., $Z_{t+1}^{cap} < Z_t^{cap}$, but now the caps are replaced by the optimal (efficient) emission level:

$$Z_{t+1}^* < Z_t^* \qquad [11]$$

It is quite easy to see that for the inequality in [11] to be reversed, technological progress (the counter clockwise rotation of the MAC curve from period t to t+1 must be larger than the impact of the change cumulative damages from emissions exceeding the self cleaning capacity of the ecosystem.

Now, assume that this is the case. The only sensible way that then generally can "preserve" dynamic cost effectiveness is to increase the emission level Z_t^* away from its static optimum. This will also be the case whenever $p_{t+1} > (1 + r)\, p_t$. Note that reducing Z_{t+1}^* would yield a similar "preservation" of the Hotelling price path, but this cannot be a welfare maximizing strategy because it would yield two welfare losses: (i) the dead weight loss from not being statically efficient in period t+1, and (ii) the additional costs associated with "excessive" emissions reductions in period t.[1]

Figure 3 illustrates the cost minimizing adjustment, i.e., $Z_t^{adj} > Z_t^*$, that preserves the Hotelling price path in the case that technological progress offsets changes in $MEC_{t+1}(Z_{t+1})$ such that the optimal equilibrium price is the same in time period t and t +1.

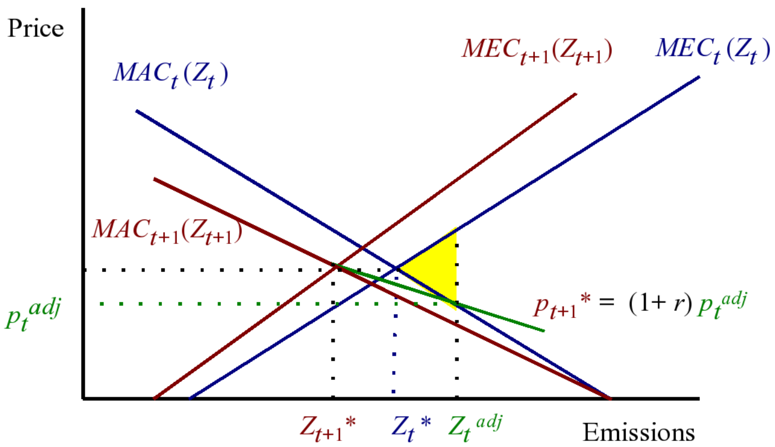

Figure 3: Adjustment in emission levels to obtain the Hotelling price path.

By increasing the emission level in period t from Z_t^* to Z_t^{adj} one is able to obtain the Hotelling price path. This adjustment comes at a cost – there are dead weight losses amounting to the yellow triangle. Hence, the benefits from being on a Hotelling price path must be compared to its costs, implying a potential trade-off between reducing the size of the dead weight losses and being off the Hotelling price path. Hence, the optimal adjustment $a_t^* < (Z_t^{adj} - Z_t^*)$.

5 Some features of deviations from the static optimum

Welfare losses, measured as a deviation from the static optimum Z_t^* often takes triangular like shapes. This implies that welfare losses grow at an increasing rate the further away from the optimum. Figure 4 provides an illustration for deviations ΔZ_t and $2\Delta Z_t$.

1 I think this will always hold, but I have yet not proven this to be the case. For now the claim made is therefore the "more modest" *generally* rather than *always*.

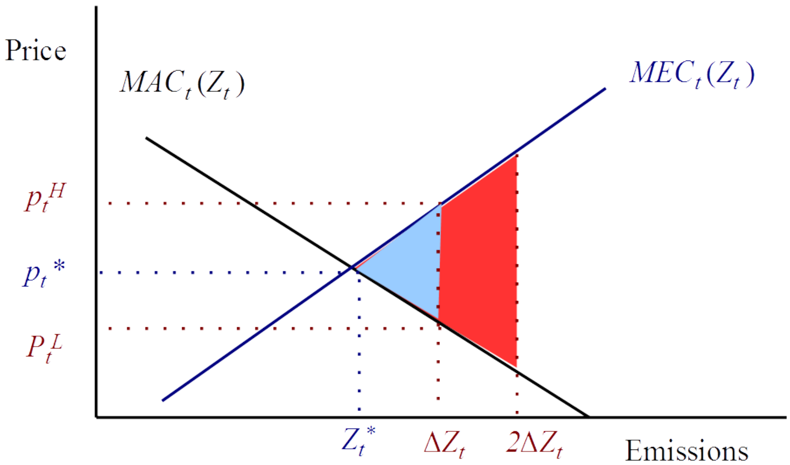

Figure 4: Increased welfare losses with increasing deviations from the optimum Z_t^*.

Figure 4 demonstrates how the welfare losses grow with increased deviations form the static optimum Z_t^*. As a matter of fact with doubling deviations (here from deviations ΔZ_t to $2\Delta Z_t$) the red area is three times as big as the blue area and the total welfare losses are four times as large for linear MAC- and MEC-curves. Reducing deviations from the sequence of static optima therefore seems to be welfare enhancing in dynamic settings. The question is what characterizes those those reductions.

6 Dynamic optimality revisited

This section is divided into three sub-sections: First, I present a two period model with full information to illustrate the principles in the trade-offs to be made. Next, I argue that under rational expectations the sequence of static optima is randomly distributed along a Hotelling price path that is adjusted for technological progress. Finally, I outline an analytical solution.

6.1 A two-period model with full information

Figure 5 displays two possible ways of maintaining time indifference, i.e., that the price in time period one equals the price in time period zero times (1+*r*).

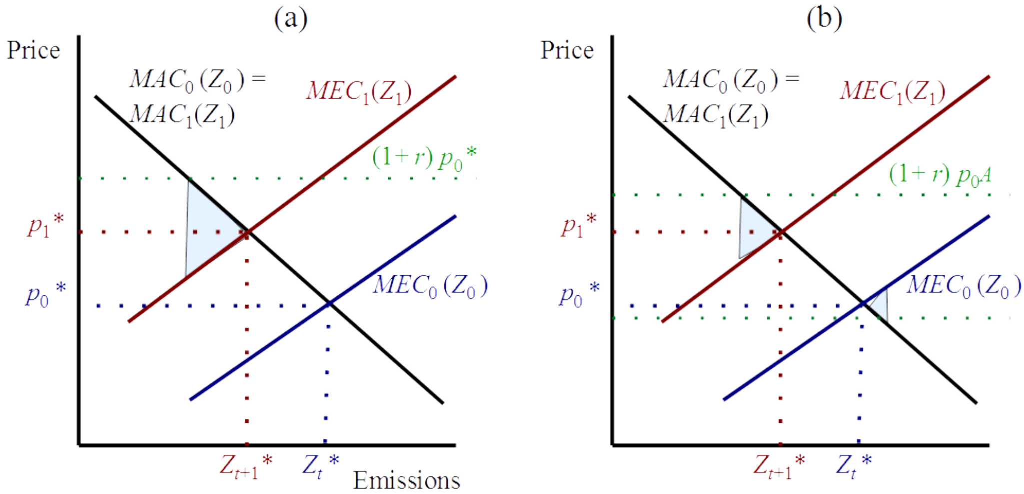

Figure 5: Two possible ways of preserving consistency in time preferences.

In panel (a) the current time period's optimum is used at a starting point. To presevere consistency in time preferences, we get quite a large welfare loss the next time period. In panel (b) time consistency is also preserved, but with far less welfare losses (the two small triangles) than the large triangle in panel (a), even when the next time period welfare losses are discounted.

Hence, this two-period example with full information, suggests that by abating less in time period zero the way Figure 5 is drawn, welfare losses can be reduced. In principle, the optimization question is allocating abatement between the two time periods. Moreover, it cannot be ruled out that welfare losses could be further reduced by relaxing the Hotelling price path condition for consistent time preferences, i.e. that

$$p_{t+1}^{x}=(1+r)\,p_{t}^{x} \qquad [12]$$

where p_{t+1}^{x} and p_{t}^{x} denote the adjusted prices from reallocating abatement decisions.

6.2 Dynamics, technological progress, stock effects, and expectations

The example in section 6.1 is not very realistic as future damages and abatement costs are unlikely to be known with certainty. Replacing actual future prices from the static optimum with current expectations, i.e.,

$$E_0[p_t]=E_0[MAC_t(Z_t^*)]=E_0[MEC_t(Z_t^*)] \qquad [13]$$

does not change the principles behind the adjustments. At the same time, the regulator cannot undo errors made in the past and readjust past emission levels. Past decisions are hence sunk costs. Therefore, the regulator needs to revise expectations about future costs as new information is made available in the next time period. These adjustments constitute an endless regress, and care needs to be taken to preserve time consistency, and to create a predictable decision environment for firms (cfr. Kydland and Prescott 1977). If these expectations are *ex-ante* rational in the sense that given current knowledge expected forecast errors are zero (Scheffrin 1985), the issues surrounding predictable decision environments are minor.

Technological progress rarely takes place at a constant rate (Ayres 1994). In stead, it usually consists of three phases – the early innovation phase where impacts on the economy is minor, main adoption phase where innovations become industrialized and economies of scale take place and progress in terms of economy wide cost savings is large, and finally a mature phase where there is little progress. This gives rise to a sigmoid path over time, where the largest uncertainty relates to when innovation takes place while industry wide adoption and the mature phases are easier to predict.

The stock effect of pollution is more complicated to capture. One reason for this is threshold effects (Nævdal 2006) which may cause unexpected changes in the marginal damages of emissions. This and other uncertain elements related to environmental damages introduces a myriad of other issues like choice of the discount rate. For long term uncertain problems, like climate, the arguments for using a lower discount rate are strong (see for example Weitzman 1988, Gollier 2002). More recent works following the Stern (2007) report provide opposing views. For example, Weitzman (2007) support the low discount rate used by Stern, while Nordhaus (2007) is quite critical towards the choice made by Stern.

6.3 *Outline of an analytical solution*

The issues mentioned in the previous section are important, but to keep the analysis more tractable, I stick to a "basic bare bones" formulation of the model. In line with the analysis in section 6.1, I assume that for the time period where decision makers are able to form reasonably well founded expectations about technological progress, changes in marginal damages from emissions, and the discount rate.

With these premises the decision maker seeks to minimize expected welfare losses from being unable to jointly solve the sequence of static optimal following from the MEC-MAC framework, and the time indifference property. For a given time period, *t*, this implies choosing the aggregate emission level to minimize the discounted sum of the deadweight losses (the triangles depicted in Figure 5) from being away from the static optima, $\{ p_t^S, Z_t^S \}$ given by

$$p_t^S = MAC_t(Z_t^S) = MEC_t(Z_t^S) \qquad [14]$$

and the discounted losses from being off the Hotelling price path, $p_t^H = p_0(1+r)^t$. These latter losses are given by the absolute value of the deviation between the conventional dynamic optimum $\{ p_t^H, Z_t^H \}$ and the revised optimum $\{ p_t^*, Z_t^* \}$, i.e.,

$$\Delta L_t(Z_t{*}^*) = |p_t^* Z_t^* - p_t^H Z_t^H| \qquad [15]$$

Let $DW_t(Z_t^*)$ denote the static dead weight losses. The policy maker's decision problem is then to choose the sequence of revised emissions $\{ Z_0^*, Z_1^*, \ldots, Z_T^* \}$ for the *T*-period planning period to minimize

$$\theta = \sum_{t=0}^{T} \beta^t [\Delta L_t(Z_t^*) + DW_t(Z_t^*)] \qquad [16]$$

where $\beta = \frac{1}{1+\delta}$ denotes the discount factor. Note that *t* = 0 belongs to the planning period.

The discount rate, δ , influences the weight of future static DW-losses, $DW_t(Z_t^*)$, and the future losses from being off the Hotelling price path [15]. Uncertainty surrounding the future loss elements in [16] suggests that this approach is more suited for adjustments in the near future, i.e., the planning horizon, *T*, may not consist of many years.

7 Concluding remarks

This note has discussed issues pertaining to *dynamic cost effectiveness* and *dynamic efficiency* in environmental emissions regulations. The main results are:

- *Dynamic cost effectiveness* entails choosing an initial emission level for the economy as a whole in the first time period, and adjusting this aggregate emission level over time so that there is no arbitrage possibilities from shifting emissions reductions between time periods. The price path that ensures that this condition is met follows from Hotelling's rule.
- *Dynamic efficiency* ideally entails having optimally set emission levels in each time period that generate prices that follow a Hotelling price path. Generally, these two conditions cannot be jointly met. The decision rule is therefore to change emission levels somewhat from the sequence of statically optimal emission levels to minimize the expected joint costs of deviating from the Hotelling price path and the dead weight efficiency losses caused by deviating from the sequence of static optimal emissions.

The adjustments that follow in the sequence of optimal emission levels $Z_0^*, Z_1^*, \ldots, Z_T^*$ for the *T*-period dynamic efficiency problem may be minor, but their implications for how we think about dynamic efficiency in the presence of externalities are important.

8 References

Ayres, R.U. (1994): Toward a non-linear dynamics of technological progress. *Journal of Economic Behavior and Organization* **24**(1):35-69.

Bergland, O. (1998): *The Economics of Stock Pollutants*. Lecture note RØ 202, Department of Economics and Social Sciences, Agricultural University of Norway, Ås, Norway.

Dasgupta, P. & G. Heal (1974): The optimal depletion of exhaustible resources. *Review of Economic Studies* **41**(1):3-28.

Gollier, C. (2002): Discounting an uncertain future. *Journal of Public Economics* **85**(2):149-166.

Hotelling, H. (1931): The Economics of Exhaustible Resources. *Journal of Political Economics* **39**(1):137-175.

Kydland, F.E. & E.C. Prescott (1977): Rules Rather than Discretion: The Inconsistency of Optimal Plans. *Journal of Political Economy* **85**(3):473-491.

Lin, C-Y, H. Meng, T.Y. Ngai, V. Oscherov & Y.H. Zhu (2008): Hotelling Revisited: Oil Prices and Endogenous Technological Progress. Unpublished working paper, Department of Agricultural and Resource Economics, University of California-Davis, Davis, CA.

Nordhaus, W.D. (2007): Review: A Review of the "Stern Review on the Economics of Climate Change". *Journal of Economic Literature* **45**(3):686-702.

Nævdal, E. (2006): Dynamic Optimisation in the Presence of Threshold Effects When the Location of the Threshold is Uncertain – With an Application to a Possible Disintegration of the Western Antarctic Ice Sheet. *Journal of Economic Dynamics and Control* **30**(7):1131-1158.

Scheffrin, S.M. (1985): *Rational Expectations*. Cambridge University Press, New York, NY.

Stern, N. (2007): *The Economics of Climate Change: The Stern Review.* Cambridge University Press, Cambridge MA.

Weitzman, M.L. (1988): Why the Far-Distant Future Should Be Discounted at Its Lowest Possible Rate. *Journal of Environmental Economics and Management* **36**(3):201-208.

Weitzman, M.L. (2003): *Income, wealth and the maximum principle*. Harvard University Press, Cambridge MA.

Weitzman, M.L. (2007): A Review of The Stern Review on the Economics of Climate change. *Journal of Economic Literature* **45**(3):703-724.