ECN 275/375 Environmental and natural resource economics
17: Renewables - forests (Perman *et al.* Ch 18)

Reading guide

Read chapter for a quick overview. Focus on sec. 18.3 (Commercial plantation forestry).

Remark on notation:

Private interest (discount) rate: Book uses i or r. This note: consistently r.

Public interest (discount) rate: Book uses i or r. This note: consistently δ.

Shadow prices on constraints: Book uses ρ (which is hard to separate from p). This note uses λ.

Forests – a brief introduction

Basic models of forest economics focus on maximizing the timber harvest value of the forests. Such models commonly assume that the forest is owned by one person, who seeks to maximize rents.

In real life, the timber price (per m^3 of timber harvested) varies depending on the following factors:

- The use of the tree product:
  - The part of the tree stem that is used for saw wood:
    - Usually collects a higher price as beams/planks can come in bigger dimensions (carry more weight).
    - Saw wood with high density (carries more weight) commands a higher price. Parts of the stem that is rotten (uncertain weight) not used/usable for saw wood.
    - The timber volume of saw wood is measured from the top diameter as this constrains the dimension of the planks that can be made from the wood stem. To see this, consider a typical tree: it is wide at the bottom and gradually tapers off towards the top of the tree.
  - The part of the stem that is used for pulp and paper:
    - Commands a lower price than saw wood.
    - Wood density not so important (fiber quality is, some rot accepted). Conifer woods (bartre) well suited for pulp/paper. Norwegian spruce (*picea abies*, Norw. gran) is the favored species in Europe ← (long fibers → possible to make thinner paper that still has the use properties demanded).
    - Volume matters for how much pulp/paper that can be made from the stem → volume measured at the middle of the stem part.
  - Lower quality stems are used for energy purposes (and collect the lowest price).
  - Special qualities can command high prices for special uses. Example: Plywood (finer): branch free birch (Norw: bjørk) and aspen (Norw: osp). Usually requires more intensive forest management, like cutting branches of the lower 5-6 m of the tree to get branch free stems).
- Trees grow slowly. Norwegian spruce and pine can be harvested for saw wood 60-70 years after planting/regeneration on productive soils – longer on less productive soils. Some tropical species like eucalyptus can be harvested after 20 years. Low density, but excellent fiber quality makes eucalyptus well suited for paper and pulp.

- Over the last 40-50 years, timber prices have dropped considerably, implying that other uses of forests have gotten increased attention, in particular hunting (where one does not need to wait 60+ years to reap benefits).
- Multiple use forestry (see sec. 18.4 for an overview)

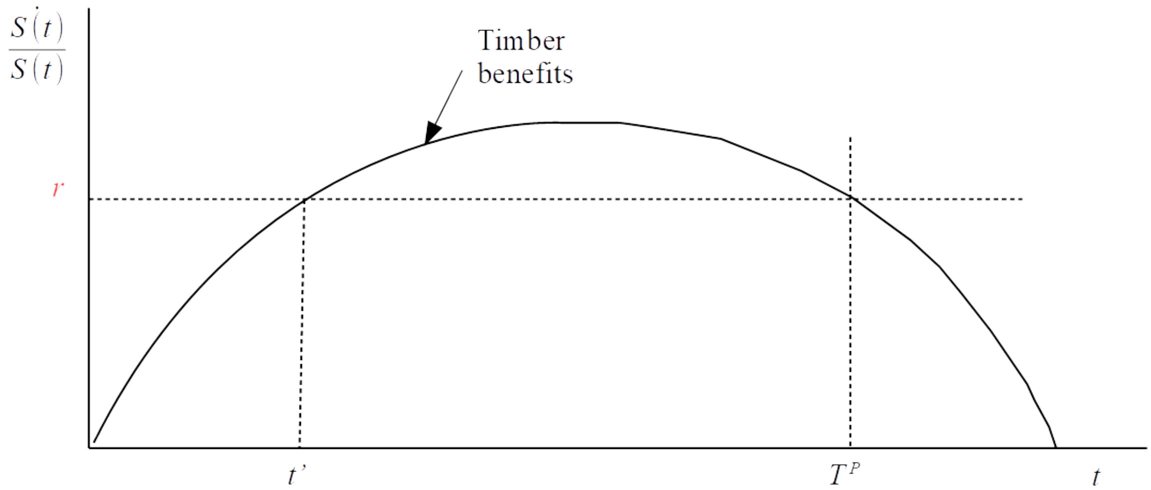
Commercial plantation (even aged stand) forestry

Introduction

To ease management and allow for clear cuts, modern commercial forestry seeks to manage stands of trees as **even aged** (benefits: easier logistics, allows for mechanized clear cuts). Even aged management involves finding the **optimal rotation age** = the age of the median tree in the stand to harvest).

The growth rate, $g(t)$,of forests as a function of initial volume is high for young forests, but gradually declines as the trees get older, i.e., $\dot{g}(t)<0$. Standing tree volume per hectare is $S(t)$. Total volume growth per time period is $\dot{S}(t)=g(t)S(t)$, so that per hectare volume grow at until some point where the growth rate $g(t)$ is so low it does not compensate for the base volume $S(t)$ it grows from.

The privately optimal rotation age is when $(P-C)\dot{S}(t)=r$, where *P* is per volume timber price, *C* is per volume harvesting costs, and *i* is the forest owner's discount rate. To simplify notation, set the net price $(P-C)=1$ (remember this step → that the real timber price is constant over time):



The optimal rotation age is *T* such that $\frac{\dot{S}(T)}{S(T)}=r \Rightarrow \dot{S}(T)=r\,S(T)$

Remark: $\dot{S}(T)=r\,'S(T)$ is also the case for *t'*, but harvesting at *t'* → the lowest return on the "tree capital" as one forfeits the growth $\dot{S}(t)>r$ for the time interval $t\in[t',T]$.

Even aged forest management – a basic single rotation model

- Net price $(P-C)=p$ is constant in real terms through the rotation.
- S_T : standing timber volume at the time of harvest
- k : replanting costs
- r : forest owner's time preference for funds (with risk free interest in the bank/capital market ← indifference between saving in timber growth or in the bank/capital market)

Objective: maximize the rents from timber harvest, choosing the rotation age, T, of the stand:

$$\begin{Bmatrix} MAX \\ T \end{Bmatrix} \pi(T) = \begin{Bmatrix} MAX \\ T \end{Bmatrix} \left(pS_T e^{-rT} - k \right)$$ (NB: no timber value until harvested → no integrals here)

FOC (differentiate in the choice variable, rotation age T):

$$\frac{d}{dT}\left(pS_T e^{-rT} - k\right) = pe^{-rT}\frac{dS}{dT} + pS_T \frac{d}{dT} e^{-rT} = 0$$

$$\Rightarrow pe^{-rT}\frac{dS}{dT} - rpS_T e^{-rT} = 0$$

$$\Rightarrow p\frac{dS}{dT} = rp\,S_T$$

$$\Rightarrow \dot{S}(t) = r\,S_T$$

T makes this equation hold. Economic interpretation: The value growth equals the interest income of the timber harvest, OR the growth rate of the trees = capital growth rate of the trees.

Multiple rotations of even aged stands – the Faustmann equation[1]

In the case of multiple rotations the solution for the single rotation is repeated for infinity. The recursive equation (18.4) gives $\Pi = \dfrac{pS_t e^{-rT} - k}{1 - e^{rT}}$ (18.7), where Π net present value of the single rotation profits π repeated for infinitely many rotations.

The Faustmann equation (Faustmann 1848) is a central equation in forest economics. It results from a version of the FOC of (18.7), and can be written in two variants:

$$\frac{p\,\dot{S}_T}{p\,S_T - k} = \frac{1}{1 - e^{rT}}$$ (18.8a: the value growth over the single rotation rents equals the capitalization factor of an activity lasting for T time periods that is repeated infinitely) OR

$$p\,\dot{S}_T = r\,pS_T + r\,\Pi$$ (18.8.b: the value growth equals the capital gains of the first rotation rents plus the capital gains of the net present value of the single period rents for infinity)

Under multiple rotations the single period rotation age goes down (see table 18.5, p. 619, for various discount rates). Main driver for this effect: the opportunity value of land – land owner profits increase when the share of time with high growth becomes larger).

[1] This brief section (18.3.2)is not exam material, but if you ever meet a theoretical forest economist, and he/she talks about the Faustmann equation, you will appear a fool if you give him/her a "blank stare". Awareness of the impacts of multiple rotations on the optimal rotation age (it goes down) is exam relevant.

Multiple use forestry

Adding non-timber value benefits, NT_t, like hunting, berry picking, biodiversity, or carbon sequestration to the optimization problem is done in the following way:

$$N_T = \int_{t=0}^{T} NT_t e^{-rt} dt$$

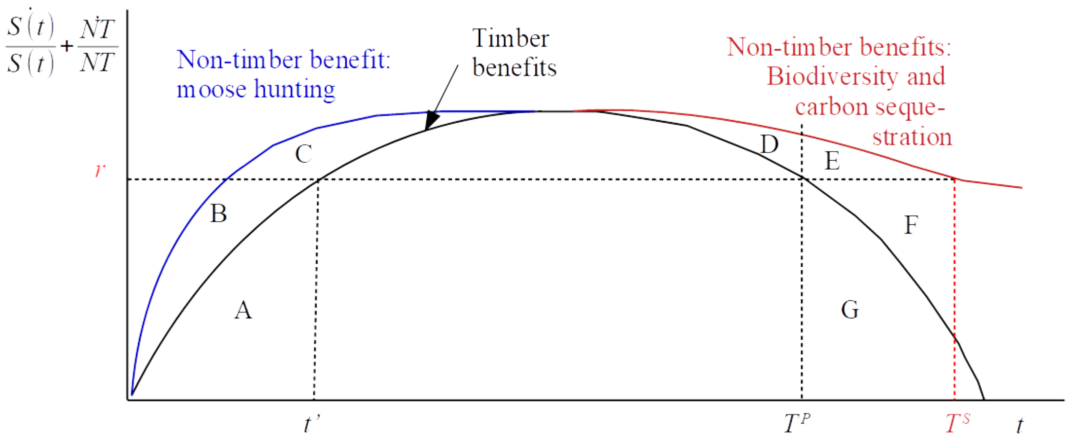
Note that timer harvests (without thinning) only provides benefits at the end of the rotation when the timber is harvested, non-timber benefits take place throughout the rotation (you can hunt every year, and carbon is sequestered every year).

The one rotation present value of timber harvests and the present value non-timber benefits, PV_1:

$$PV_1 = pS_T e^{-rT} - k + N_T = pS_T e^{-rT} - k + \int_{t=0}^{T} NT_t e^{-rt} dt$$

Depending upon the time profile of the non-timber benefits throughout the rotation, the optimal rotation length could change in various ways. Two examples to illustrate this (corresponding graph on the next page):

- Moose hunting (where benefits are early in the rotation, in particular on clear cuts where the occurrence of herbs and other moose menu items increase the moose population and hence hunting benefits). Note that this does not change the optimal rotation age in the single rotation problem as the $\dot{S}/S$ curve does not change where it crosses the interest rate line r. In the multiple period case this shortens the rotation age more than in the standard Faustmann case because of the "moose rents" (= added area under the blue curve and above the black curve).
- Biodiversity that usually is associated with old growth forest and even forests so old that the amount of dead wood becomes large – in this example, the biodiversity impacts are valued so high that the optimal rotation age increases beyond the period where the timber value grows to T_{+NT}).



Exercises

See exercises on the course web page.

Discussion topics

Single rotation management

1. In the basic timber harvest model, the net timber price, $p = P - C$, is assumed to be constant throughout the rotation. Suppose the net timber price increased through the rotation. How would that affect the optimal rotation age? Justify your answer.
2. What are the impacts of an increase in the planting/regeneration costs on the optimal rotation age? Explain your answer. (Hint: look at the equation for the optimal rotation age)

Non-timber benefits

1. What other factors than biodiversity associated with old-growth forest could make the optimal rotation age increase as illustrated in the figure on the previous page? Explain briefly your reasoning.
2. How would you make land owners voluntarily extend the rotation age from T to T_{+NT} in the biodiversity case?
3. What is the impact of the non-timber benefits from moose hunting on the optimal rotation age under single and multiple rotations? When explaining your answer, emphasize the difference between the single and multiple rotation cases.