ECN 275/375: Forests

– discussion questions single rotation, clarifying note

This note is also revised with the notation used in lectures 15-17:

- Private interest (discount) rate: Book uses i or r. This note: consistently r
- Public interest (discount) rate: Book uses i or r. This note: consistently δ
- Shadow prices on constraints: Book uses ρ (which is hard to separate from p). This note uses λ

Main points in this note:

1. In the basic timber harvest model, the net timber price, $p=P-C$, is assumed to be constant throughout the rotation. Suppose the net timber price increases through the rotation. How would that affect the optimal rotation age? Justify your answer.

   Answer: Suppose the net price grows at the rate ρ (not to be confused with the book's notation) through the rotation period. The revised profit maximization problem now becomes:

   $$\left\{\begin{matrix} MAX \\ T \end{matrix}\right\}\pi(T)=\left\{\begin{matrix} MAX \\ T \end{matrix}\right\}\left(p\,e^{\rho T}S_T e^{-rT}-k\right)=\left\{\begin{matrix} MAX \\ T \end{matrix}\right\}\left(p\,S_T e^{(\rho-r)T}-k\right)$$

   which gives the solution $\frac{\dot{S}}{S_T}=r-\rho<r$ as $\rho > 0$. This implies the optimal rotation age goes up as the net discount rate $(r-\rho)$ becomes lower.

   Remark: A reasonable result as as the value of the forest capital increases. Still my remark in class about price oscillations around T are valid, i.e., forest owners view the forcasted the short term net price development when they decide to adjust T somewhat.

2. What are the impacts of an increase in the planting/regeneration costs on the optimal rotation

   Answer: k (see equation in (1)) ia a constant with no T (the choice variable we differentiate with). Hence k vanishes in the FOC, and we get no impact of changed planting costs in the optimal rotation age.

   Remark: The decision to plant or not affects the rotation age in the following way: *No planting*, and it takes more time for the preferred tree species to establish itself → the rotation age increases compared to with *planting*.

   Let T and U denote the optimal rotation ages with and without planting such that $T < U$. We now need to compare:

   with planting: $$\left\{\begin{matrix} MAX \\ T \end{matrix}\right\}\pi(T)=\left\{\begin{matrix} MAX \\ T \end{matrix}\right\}\left(p\,S_T e^{-rT}-k\right)=\left\{\begin{matrix} MAX \\ T \end{matrix}\right\}\left(p\,S_T e^{-rT}-k\right)$$

   without planting: $$\left\{\begin{matrix} MAX \\ U \end{matrix}\right\}\pi(U)=\left\{\begin{matrix} MAX \\ U \end{matrix}\right\}\left(p\,S_T e^{-rU}-0\right)=\left\{\begin{matrix} MAX \\ U \end{matrix}\right\}\left(p\,S_U e^{-rU}\right)$$

   The prolonged rotation period without planting ($U > T$) → that the discounted value of the timber harvest without planting is less than with planting. If this difference is larger than the saved planting costs, it is optimal to plant. In the opposite situation, it is optimal not to plant.