ECN 275/375 Environmental and natural resource economics 14: Stock pollutants (Perman *et al.* Ch 16)

Reading guide

Read entire chapter for an overview. Focus on sec. 16.1 (model framework). Read sec. 16.4 to see how this model framework is applied. As before, solving Hamiltonians not part of the curriculum for ECN 275/375.

Key concept: steady state = the condition when the time derivatives of the pollution or accumulation constraints for emissions are zero. See section 16.3 for a discussion.

Basic model framework (16.1)

The model in sec. 16.1 operates with an index of environmental pressures, *E*. Welfare $U(C,E)$ is increasing in consumption, $U_C(C,E)>0$, and decreasing in the envinronmental pressure, $U_E(C,E)<0$. Environmental pressure, $E(R,A)$, has two environmentally related variables:

1. *R* [resource extraction as in ch. 14 – think of *R* as oil].
2. *A* [accumulation of a byproduct from the extraction and use of *R*, which leads to an ambient pressure – hence *A* is used for this variable].

Environmental pressure increases with both resource use and ambient levels (of accumulated pollutants) → $E_R(R,A)>0$ and $E_A(R,A)>0$.

The costs of extracting the non-renewable resource is given by $\Gamma(R)$, where $\Gamma_R>0$.

The welfare impacts depicted in $U(C,E)$ can be:

- direct consumer externalities (like polluted air affecting health) → the parts of the environmental index is directly included in the welfare function, i.e., $U(C,E(R,A))$, or
- indirect production externality (through reduced production and hence higher prices for consumer goods) →
    - the utility function simplifies to $U(C)$
    - the production function is rewritten to $Q(R,K,E(R,A))$
    - remark: if a clear consumer or producer externality, I recommend to choose the simplest possible approach as this reduces the number of partial derivatives to keep track of.

Resource-stock relationship (16.1.3)

In the model setup in the book, emissions are a function of resource use: $M(R), M_R>0$

Assuming a constant self cleaning (decay) rate, *α*, from the accumulated pollutant, *A*, the change in ambient pollution levels are $\dot{A}=M(R_t)-\alpha A_t$, where $\dot{A}>0\,\forall\, R_t, A_t : M(R_t>\alpha A_t)$ and $\dot{A}<0\,\forall\, R_t, A_t : M(R_t<\alpha A_t)$. An important implication of such effects is that the dynamics along the system may change on either side of the equilibrium condition $\dot{A}=0\,\forall\, R_t, A_t : M(R_t=\alpha A_t)$. (fig. 16.5 on a phase diagram shows this effect along the line $\dot{A}=0$).

Note that $\alpha=0 \Rightarrow \dot{A}=M(R)$, i.e., only depends on emissions → a risk that emissions accumulate forever unless emissions are zero, i.e., $M(R)=0$.

Clean-up expenditures (16.1.4)

For stock pollutants there is an additional policy alternative: cleaning up the accumulated pollutants at the cost *V*. Normalizing prices to one (a usual "trick") to reduce the complexity of already complicated models, gives:

$Q \equiv \dot{K} + C + \Gamma + V$ as the additional policy variable *V* gives a change in the capital stock, *K*. (Remark: Using the "trick" of setting the price to one, makes it possible to eliminate one variable → simplifies what already may appear as a complicated setup, but really is not so bad = lot of economic intuition).

Clean up expenditures, *V*, transform to reduced accumulations of emissions via $F(V)$ where $F_V(V) > 0$. The differential equation for the pollution stock then gets an additional term:

$\dot{A} = M(R) - \alpha A - F(V)$ [αA = natural decay (self cleaning), $F(V)$ = clean up]

Complete stock pollution problem with natural decay and clean-up (16.1.5)

C, R, and *V* are the control (choice) variables for the dynamic welfare optimization problem (in this exposition *r* is used in stead of ρ for the discount rate) :

$$\begin{Bmatrix} MAX \\ C, R, V \end{Bmatrix} W = \begin{Bmatrix} MAX \\ C, R, V \end{Bmatrix} \int_0^\infty U(C, E(R, A)) e^{-rt} dt$$

s.t.

$\dot{S} = -R_t$ (resource stock change)

$\dot{A} = M(R) - \alpha A - F(V)$ (pollution accumulation change)

$\dot{K} = Q(K_t, R_t, E(R_t, A_t)) - C_t - \Gamma(R_t) - V_t$ (capital change)

3 constraints give three shadow prices (co-state variables OR Lagrangian multipliers) in the current value Hamiltonian:

$H = U(C, E(R, A))$

$+P_t(-R_t)$ (resource constraint: as before = inserting a resource price as the shadow price)

$+\lambda_t(M(R_t) - \alpha A_t - F(V_t))$ (pollution accumulation constraint)

$+\omega_t(Q(K_t, R_t, E(R_t, A_t)) - C_t - \Gamma(R_t) - V_t)$ (capital constraint into production)

Remarks: Setting up the current value Hamiltonian makes life a whole lot easier:

1. We have gotten rid the integral and the discount rate in the objective function (it will come back later in characterizing the optimal solution).
2. The rest looks familiar to us with the Lagrangian multipliers we know from before).
3. We will not solve for the Hamiltonian – if commenting on solutions for dynamic problems on the exam, you will be given the solution).
4. This is quite a complicated model with both production and consumption externalities.

The optimal solution (without time subscripts) + comments

$U_C = \omega$ (the marginal utility of consumption equals the shadow price on capital: Reason – in optimum one is indifferent between consumption and constraining capital)

$P = U_E E_R + \omega Q_K + \omega Q_E E_R - \omega \Gamma_R + \lambda E_R$ (the resource price captures all externalities in optimum – if it did not, we are not in an optimum – typo in the book: M_R should be E_R as here)

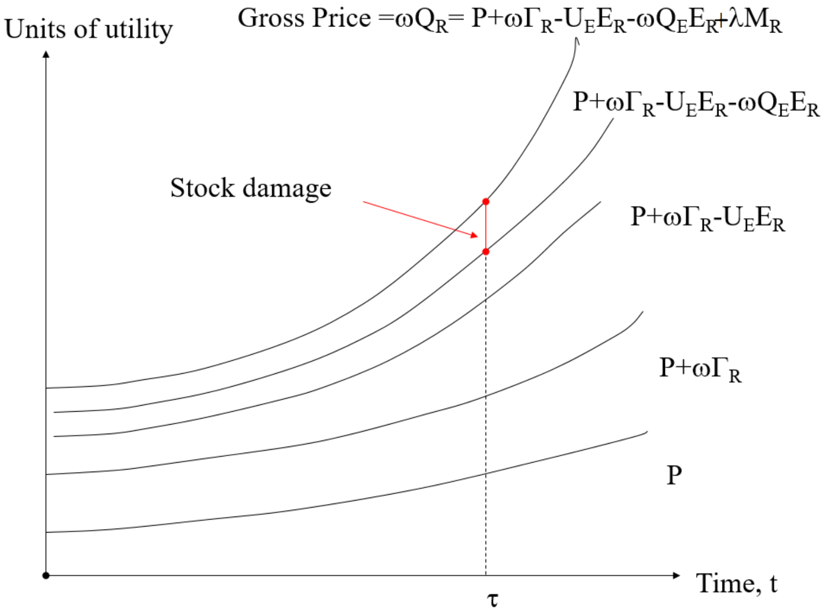
$\omega_t = -\lambda_t F_V$ (the shadow price on capital in optimum equals the marginal effect of clean-up times the size of the Lagrangian multiplier for the pollution accumulation equation = static efficiency condition for the cleanup costs)

$\dot{P}_t = r P_t$ (this is another way of writing Hotelling's rule = the change in the resource price equals the discount rate *r* times the shadow price on the resource constraint (which again equals the resource price in optimum)

$\dot{\omega}_t = r\,\omega_t - Q_K \omega_t$ (the time change in the shadow price on the capital constraint equals the discount rate *r* times the shadow price on the capital constraint (see the similarity to the price change equation above))

$\dot{\lambda}_t = r\,\lambda_t + \alpha\,\lambda_t - U_E E_A - \omega_t Q_E E_A$ (the time change in the shadow price on the pollution accumulation constraint equals the rate *r* times the shadow price on the pollution constraint (see the similarity to the price change equation above)) + the natural decay rate α times the shadow price on the pollution constraint less the marginal impact on utility of pollution accumulation less shadow price on the capital constraint times the marginal impact on production from the stock pollutant

Figures 16.2 (right) and 16.3 show the interpretation in terms of optimal time paths for the variables of the model (time indexes dropped).



(Remark: It may appear the impacts of the consumption externality, $U_E E_R$, and the production externality, $\omega Q_E E_R$, have negative impacts on corrected price. Recall that the impacts of the utility and production, U_E and Q_E, are negative → the effects on the tax (fig. 16.3) are as expected. To avoid this, frame environmental quality positively (makes it easier to interpret equations).

There are three taxes needed to correct this externality. To see that rewrite the price equation:

$$P_t = U_E E_R + \omega_t Q_K + \omega_t Q_E E_R - \omega_t \Gamma_R + \lambda_t M_R$$

to:

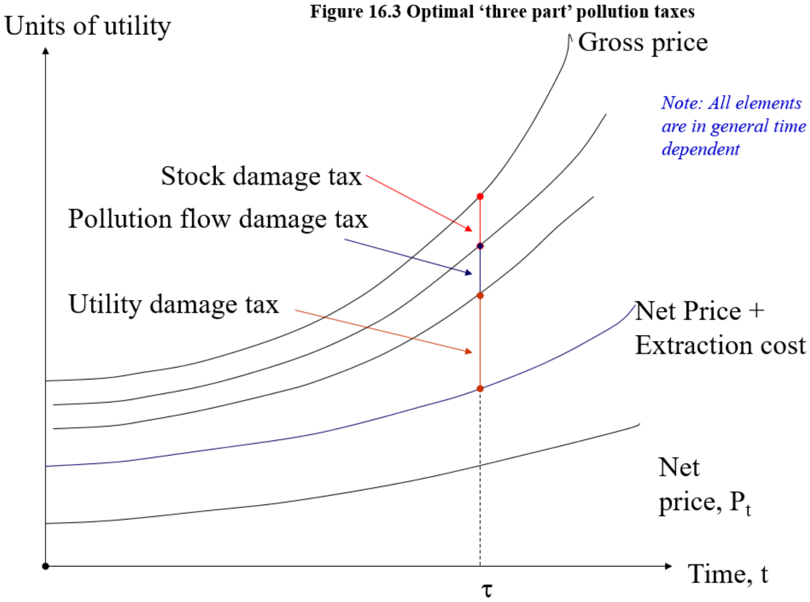
$$\omega_t Q_K = P_t + \omega_t \Gamma - U_E E_R - \omega_t Q_E E_R - \lambda_t M_R$$

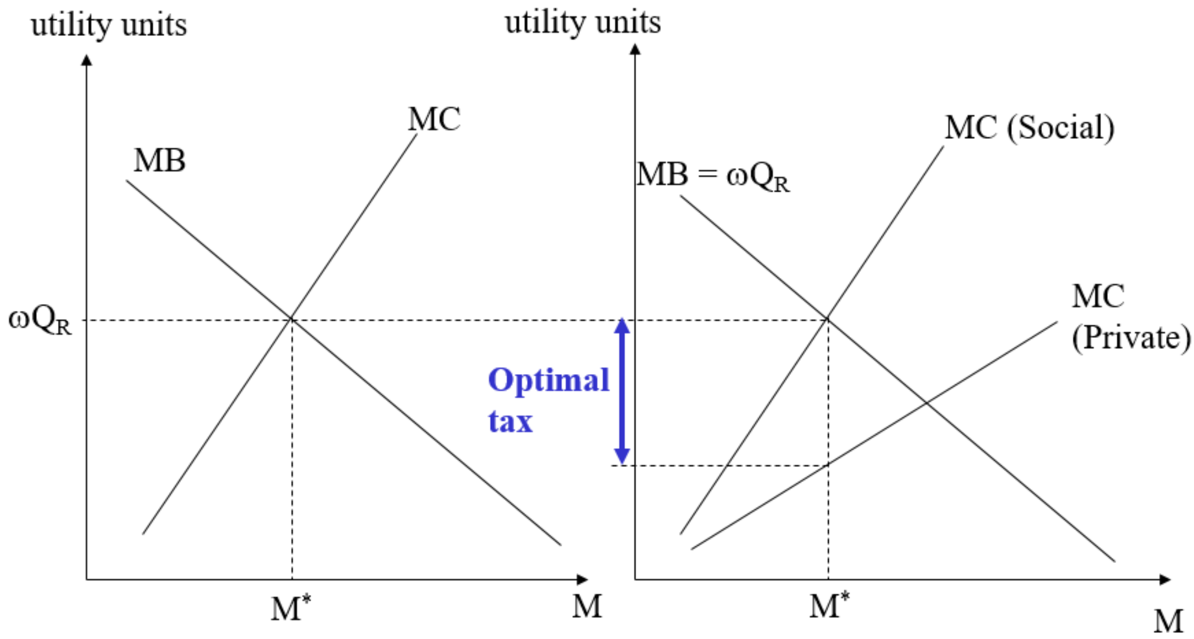
utility damage tax

pollution flow damage tax

stock damage tax

Figure 16.3 Optimal 'three part' pollution taxes





Notes: $MC = MC\ (\text{social}) = P + \omega\Gamma_R - U_E E_R - \omega Q_E E_R - \lambda M_R$

$MC\ (\text{private}) = P + \omega\Gamma_R$

$\text{Optimal tax} = - U_E E_R - \omega Q_E E_R - \lambda M_R$

$MB = \omega Q_R$

Figure 16.4 Optimal taxes and the wedge between private and social costs.

Exercises

Focuses on understanding the (net) emission constraint, and applying the four quadrants graph on the stock pollution problem, where the resource constraint in the 3^{rd} quadrant is replaced by allowed (net) emissions.

Discussion topics

1. Consider the model at the beginning of section 16.1.5. How would you rewrite (simplify) the model if there is no production externality, i.e., only a consumption externality?
2. In section 16.4 on steady state, Perman *et al.* argue that the steady state is irrelevant for the climate change as long as fossil fuels are still extracted. How can you soften this conclusion (hint: how to include carbon sequestration in the model at the beginning of section 16.1.5).