ECN 275/375 Environmental and natural resource economics 14: Stock pollutants – in perspective

Climate gas emissions damage is a typical case for this model setup (think of this as we have a carbon budget in the atmosphere that we gradually empty – in this case, fill up).

Key concept: steady state = the condition when the time derivatives of the pollution or accumulation constraints are zero. See section 16.3 for a discussion.

See lecture note 14 (http://arken.nmbu.no/~eiriro/ecn275/lectures/lec-14.pdf) for an explanation of terms – this note starts by comparing the stock pollutant model with the model in 13 on *Optimal resource extraction of non-renewables* (http://arken.nmbu.no/~eiriro/ecn275/lectures/lec-13.pdf). Note the similarity in the model formulations. Parts of this note is "cut and paste" from the original Lecture 14 note.

Compared to the model in lecture 13, there is an additional policy alternative: cleaning up the accumulated pollutants at the cost *V* (normalizing product prices to one (a usual "trick") to reduce the complexity of already complicated models): $Q \equiv \dot{K} + C + \Gamma + V$ as the additional policy expenditure variable *V* gives a change in the capital stock, *K*.

Clean up expenditures *V* transform to reduced accumulations of emissions via $F(V)$ where $F_V(V) > 0$. The differential equation for the pollution stock then gets an additional term:

$\dot{A} = M(R) - \alpha A - F(V)$ [αA = natural decay (self cleaning), $F(V)$ = clean up]

C (consumption) *R* (resource use), and *V* (clean up expenditures or in the climate case: reduced fossil fuel use or industrial carbon sequestration) are the control (choice) variables for the dynamic welfare optimization problem where *r* is the discount rate:

$$\begin{Bmatrix} MAX \\ C_t, R_t, V_t \end{Bmatrix} \int_0^\infty U(C_t, E(R_t, A_t)) e^{-rt} dt$$

s.t.

$\dot{S} = -R_t$ (resource stock change)

$\dot{A} = M(R) - \alpha A - F(V)$ (pollution accumulation change)

$\dot{K} = Q(K_t, R_t, E(R_t, A_t)) - C_t - \Gamma(R_t) - V_t$ (capital change)

Compare this with the "cake eating" model of lecture 13 (reformulated from 2 periods to infinite continuous time for easier comparison) where consumption is given by resource use (cake eaten):

$$\begin{Bmatrix} MAX \\ C_t \end{Bmatrix} \int_0^\infty U(C_t) e^{-rt} dt$$

s.t.

$\dot{S} = -R_t$ (resource stock change)

$C_t = R_t$ (consumption equals resource use – added to make model formulations more equal)

The cake eating model has one constraint as we directly can use $C_t = R_t$ to rewrite the model as:

$$\begin{Bmatrix} MAX \\ R_t \end{Bmatrix} \int_0^\infty U(R_t) e^{-rt} dt$$

s.t.

$\dot{S}=-R_t$ (resource stock change) and $\bar{S}=\int_0^\infty R_t$ (dno change the problem in terms of constraints)

The current value Hamiltonian (extended Lagrange) formulation of the "cake eating" model then becomes:

$H=U(R_t)$

$+P_t(-R_t)$ (resource constraint: as before = inserting a resource price as the shadow price)

The lecutre 14 model has two additional constraints (in red) to give three shadow prices (co-state variables OR Lagrangian multipliers) in the current value Hamiltonian:

$H=U(C_t, E(R_t, A_t))$

$+P_t(-R_t)$ (resource constraint: as before = inserting a resource price as the shadow price)

$+\lambda_t(M(R_t)-\alpha A_t-F(V_t))$ (pollution accumulation constraint)

$+\omega_t(Q(K_t, R_t, E(R_t, A_t))-C_t-\Gamma(R_t)-V_t)$ (capital constraint into production)

The optimal solution of the stock externality model (without time subscripts) + comments

$U_C=\omega$ (the marginal utility of consumption equals the shadow price on capital: Reason – in optimum one is indifferent between consumption and constraining capital)

$P=U_E E_R+\omega Q_K+\omega Q_E E_R-\omega\Gamma_R+\lambda E_R$ (the resource price captures all externalities in optimum – if it did not, we are not in an optimum)

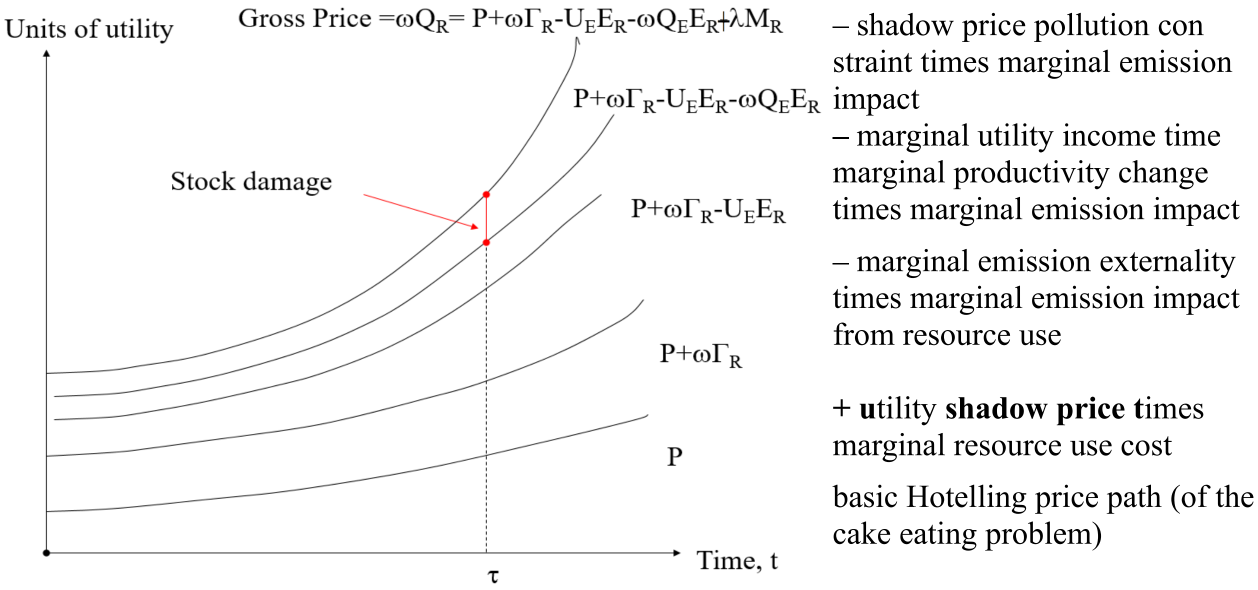
$\omega_t=-\lambda_t F_V$ (the shadow price on capital in optimum equals the marginal effect of clean-up times the size of the Lagrangian multiplier for the pollution accumulation equation = static efficiency condition for the cleanup costs)

$\dot{P}_t=rP_t$ (this is another way of writing Hotelling's rule = the change in the resource price equals the discount rate *r* times the shadow price on the resource constraint (which again equals the resource price in optimum)

$\dot{\omega}_t=r\omega_t-Q_K\omega_t$ (the time change in the shadow price on the capital constraint equals the discount rate *r* times the shadow price on the capital constraint (see the similarity to the price change equation above))

$\dot{\lambda}_t=r\lambda_t+\alpha\lambda_t-U_E E_A-\omega_t Q_E E_A$ (the time change in the shadow price on the pollution accumulation constraint equals the rate *r* times the shadow price on the pollution constraint (see the similarity to the price change equation above)) + the natural decay rate α times the shadow price on the pollution constraint less the marginal impact on utility of pollution accumulation less shadow price on the capital constraint times the marginal impact on production from the stock pollutant

Figures next page show the development of the prices for the lecture 14 model:



Note that the bottom line in this figure is the solution to the "cake eating" problem in Lecture 13, so our expansion of the problem adds a lot more.

There are three taxes needed to correct this externality. To see that rewrite the price equation:

$$P_t = U_E E_R + \omega_t Q_K + \omega_t Q_E E_R - \omega_t \Gamma_R + \lambda_t M_R$$

to:

$$\omega_t Q_K = P_t + \omega_t \Gamma - U_E E_R - \omega_t Q_E E_R - \lambda_t M_R$$

utility damage tax

pollution flow damage tax

stock damage tax

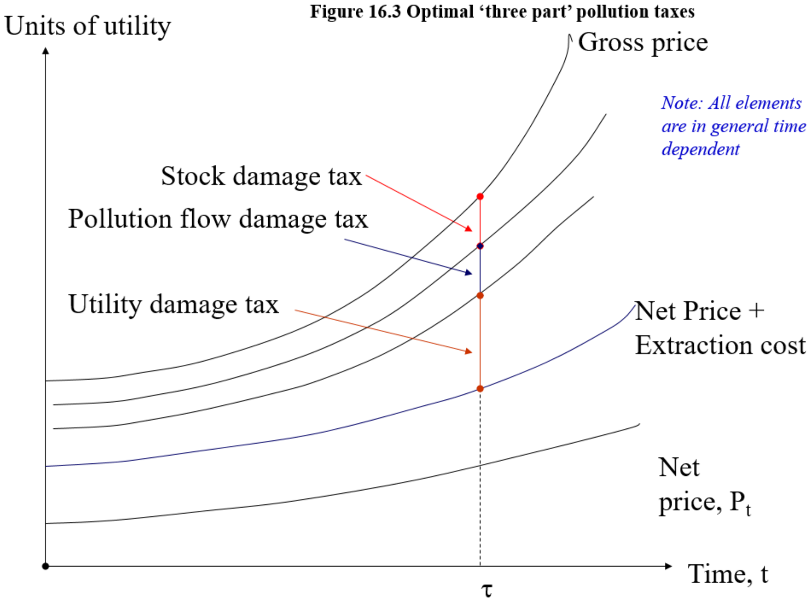


Figure 16.3 Optimal 'three part' pollution taxes

The stock pollutant model can be further rewritten by adding a ceiling of accumulated emissions, $\bar{S}$ for example the amount of accumulated emissions allowed to stay within the 2 degrees target of the Paris agreement by adding the following constraint:

$$\bar{S} = \int_0^\infty E(R_t, A_t)$$

where $E(R_t, A_t)$ is the emissions as a function of resource (in the climate example fossil fuel) use corrected for the abatement efforts.

This change in the model enables us to use the four-quadrants graph for non-renewable resources if we want to graphically illustrate what happens as the accumulated emissions constrain changes, for example from tightening or relaxing the 2 degrees target of the Paris agreement.

Further comments

In the top graph on the previous page, the various price paths are associated to the (Lagrangian/-Hamiltonian) multipliers. It starts with the bacic Hotelling price path. As we add the new elements from the model: impacts on production and hence consumption (the only place in this model where agents derive utility), etc, we get adjustments in the price paths.

In the bottom graph we sort the various parts of the solution according to the corrective policy instruments (taxes) we have to our disposal:

- *a utility damage tax* from the impacts on consumption from reduced production,
- *a pollution flow tax* to correct for the direct disutility from emissions, and
- *a stock damage tax* to correct for the stock damages.

Note that the two bottom lines, the basic Hotelling price path, and the Hotelling price path when we add increased costs of extracting the resource as it becomes more scarce, are not external effects that need corrective measures.

The bottom graph hence makes it (more) clear that we need various types of policies (here: taxes) to correct for the type of damage: *onto the externality on production* which negatively affects consumption and hence utility, *onto pollution flows* which cause direct disutlity, and *onto stock damages*. The need for separate instruments for these three externalities goes back to Tinbergen: one instrument per type of policy objective we have to secure that each of these objectives are met. Note that if we are unable to design an appropriate policy for an objective or the objective is not met because of *Pareto irrelevance*, we abstain from correcting that type of externality.