ECN 275/375 Environmental and natural resource economics
13: Efficient use of non-renewables
(Perman *et al.* Ch 15)

Reading guide

Read entire chapter for an overview. Emphasis on the 2-period model (section 15.1) and the graphical analysis following the format of Fig. 5.3 (p. 520). Parts of the chapter is quite technical, and the simpler 2-period model + the graph format improves understanding → better analysis more exactly: the analysis becomes relevant).

Regarding the multi-period model (section 15.2) try to see the similarities between the 2-period and *N*-period problem. Note that the *N*-period problem is conveniently framed as a continuous time problem. (Discrete time problems can be modeled using dynamic programming, but modeling in continuous time is so much easier)

In addition, not the discussion on situations where Hotelling's rule ($P_t = P_0 e^{\delta t}$ in continuous time OR $P_t = P_0 (1+\delta)^t$ in discrete time) is expected not to hold).

A simple 2-period model (sec. 15.1)

Decision problem: How much to use of a fixed a fixed resource (the "cake" of size $\bar{S}_t$) in 2 periods when there is discounting.

Resource use (extraction): R_t for t = {0, 1}.

Simple and identical linear demand function (to keep algebra simple) for the resource (*R* replaces *Q* from ordinary demand theory) for two time periods: $R_t = \frac{a - P_t}{b} \Rightarrow$ inverse demand $P_t = a - b R_t$.

Fig. 15.1 illustrates the inverse demand (consumer surplus is now the shaded are to the left of the chosen quantity to extract). Mathematically

$$B(R_t) = \int_0^{R_t} (a - b R_t) dt = a R_t - \frac{b}{2} R_t^2$$

Total extraction costs assumed linear in resource extraction: $C_t = c R_t \Rightarrow C_t{}' = c$

Total net social benefits in time *t* (NSB_t) from resource extraction R_t :

$NSB_t =$ $U_t(R_t) = B(R_t) - C_t{}' = \int_0^{R_t} (a - b R_t - c) dt = a R_t - \frac{b}{2} R_t^2 - c R_t$

2-period discounting when 1st period is time = 0 and the discount rate is *r* (the book uses ρ)

$$W = \sum_{t=0}^{1} \left(\frac{1}{1+r} \right)^t U_t = \left(\frac{1}{1+r} \right)^0 U_0 + \left(\frac{1}{1+r} \right)^1 U_1 = U_0 + \frac{1}{1+r} U_1$$

The book replaces NSB with *U* (which for some steps makes things simpler). Decision problem:

$$\left\{ \begin{matrix} MAX \\ R_0, R_1 \end{matrix} \right\} W = \left\{ \begin{matrix} MAX \\ R_0, R_1 \end{matrix} \right\} \left(U_0(R_0) + \frac{1}{1+r} U_1(R_1) \right) \text{ subject to: } \bar{S} = R_0 + R_1 \text{ (the resource constraint)}$$

Lagrangian:

$$\mathscr{L} = U_0(R_0) + \frac{1}{1+r} U_1(R_1) + \lambda(\bar{S} - R_0 - R_1)$$

$$= aR_0 - \frac{b}{2}R_0^2 - cR_0 + \frac{1}{1+r}\left(aR_1 - \frac{b}{2}R_1^2 - cR_1\right) + \lambda(\bar{S} - R_0 - R_1)$$

$$= aR_0 - \frac{b}{2}R_0^2 - cR_0 + \frac{1}{1+r}\left(aR_1 - \frac{b}{2}R_1^2 - cR_1\right) + \lambda(\bar{S} - R_0 - R_1)$$

First order conditions:

(1) $\frac{\partial \mathscr{L}}{\partial R_0} = a - bR_0 - c - \lambda = 0$

(2) $\frac{\partial \mathscr{L}}{\partial R_1} = \left(\frac{1}{1+r}\right)(a - b + R_1 - c) - \lambda = 0$

(3) $\frac{\partial \mathscr{L}}{\partial \lambda} = \bar{S} - R_0 - R_1 = 0$

(1) and (2) both equal zero → set (1) equal to (2) (gets rid of λ)

$$a - bR_0 - c - \lambda = \left(\frac{1}{1+r}\right)(a - bR_1 - c) - \lambda$$

⇓ use $P_t = a - bR_t$

$$P_0 - c = \left(\frac{1}{1+r}\right)(P_1 - c)$$

⇓ remark: $P_t - c$ is the net price = resource rent

$$P_1 - c = (1+r)(P_0 - c) \quad \text{(which is Hotelling's rule)}$$

Remark: Adding one more time period to the 2-period problem adds one more equation like (2), and it quickly gets messy. This is where Hamiltonians enter (solving not part of ECN 275 exam curriculum).

Multi-period problems

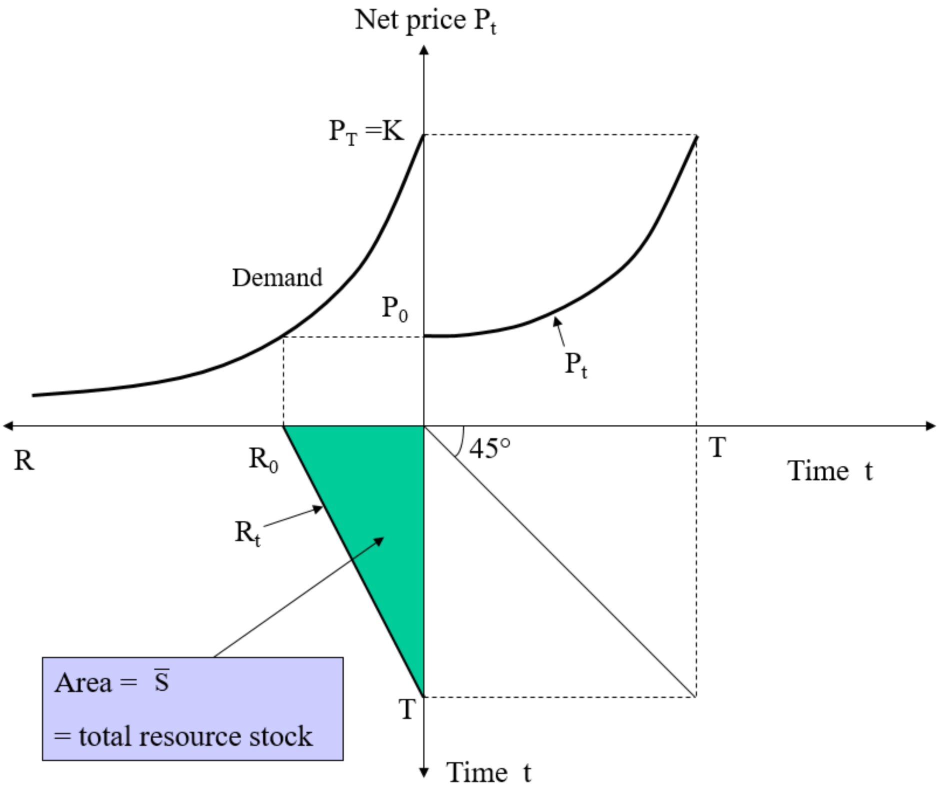
Main changes compared to 2-period problems

1. More conveniently modeled in continuous time → optimal control theory and Hamiltonians
2. The resource constraint
    1. Discrete time 2-period: $\bar{S} = R_0 + R_1 = \sum_{t=0}^{1} R_t$ T periods: $\bar{S} = R_0 + \ldots + R_{T-1} = \sum_{t=0}^{T-1} R_t$
       (* here we often simplify and write the upper time limit as T in stead of T–1)
    2. Continuous 0-1 time: $\bar{S} = \int_0^1 R_t\,dt$ T time: $\bar{S} = \int_0^T R_t\,dt$
3. The objective function
    1. Discrete time 2-period: $W = \sum_{t=0}^{1}\left(\frac{1}{1+r}\right)^t U_t$ T periods: $W = \sum_{t=0}^{T-1}\left(\frac{1}{1+r}\right)^t U_t$ *
    2. Continuous 0-1 time: $W = \int_0^1 U_t e^{-rt}\,dt$ T time: $W = \int_0^T U_t e^{-rt}\,dt$

Note: Same time boundaries used on the resource constraint and the objective function.

Graphical description of the resource use solutions

Figure 15.3 (p. 520) the key to understanding how to go about solving such problems



Comments:

1. The 45 degree line (lower right hand quadrant) is a frequently used "trick" to couple multiple graphs.
2. *K* is the choke price = price that sets demanded quantity to zero, for example the price when the backstop technology replaces the resource. Recall that the Hotelling (net) price path includes extraction costs (the *c* in the models in this chapter). Any monopolist or cartel now needs to set its initial extraction R_0 in such a way that rents are maximized (i.e., the area under the Hotelling price path for whatever duration for extraction is maximized). Then stop time becomes a choice variable in the decision problem, and we get:

   $\begin{Bmatrix} MAX \\ R_0, T' \end{Bmatrix} W = \int_0^{T'} U(R_t) e^{-rt} dt$ as the monopolist's choice of R_0 influences T' and the initial price, P_0, and therefore also the Hotelling price path

   $\bar{S} \geq \int_0^{T'} R_t dt$ note "≥" constraint → allows not extracting the full resource amount unless total resource extraction is consistent with objective function

3. When using the graph for analysis, start asking what is the key difference(s) from the basic problem = what change(s). Some examples:
   1. If the time period to be analyzed (T) increases, and the resource stock remains unchanged (area of green triangle the same), R_0 must move to the right (become smaller).
   2. If demand increases (outward demand curve shift in upper left hand quadrant) and the resource stock remains unchanged, (i) initial extraction R_0 increases → time for which the resource exists, must decline. (ii) the initial price P_0 in the upper right hand increases – from Hotelling's rule: price path shifts out but still obeying $P_t = P_0 e^{rt}$ ($r = \rho$ in book)

Comments on Hotelling's rule

Hotelling's rule, $P_t = P_0 e^{rt}$, is built on the premise that production or consumption can be moved between time periods to enable *no arbitrage* between periods. When resource stocks are finite, a scarcity factor, *s*, may be added to the discount rate to extend the "life" of the resource, i.e., $P_t = P_0 e^{(r+s)t}$. The size of the scarcity factor increases with higher concerns on the welfare impacts of (too?) low resource availability in the future. In the book, the scarcity factor is often included in the interest rate. One rationale for this is that the interest rate, *r*, here is not a market rate, but our time preference. Ideally, adding a scarcity factor would make it easier to follow the mathematics.

Some relevant cases with adjustments:

1. New reserves (like oil are found) at some discrete time intervals (see fig. 15.8) → after each new major finding, there are time segments from the time of the new finding where the "Hotelling price path" continues (until a new reserve is found, and the price drops – story repeated). See also fig. 15.7 for changes in "lifetime" and initial price change
2. The interest rate, *r*, increases → initial demand falls (→ P_0 falls) + steeper price path → "life of resource" could be extended (see fig. 15.6) (but eventually higher price → stronger incentives for getting backstop technology in place → "life of resource" increases, but its importance for welfare declines).

Exercises

Focuses on Hotelling's rule and the Hotelling price path, and the graphical representation of resource extraction (four quadrants graph).

Discussion topics

Use figure 15.3 (top of this page) as a starting premise.

1. Suppose that estimated oil reserves decline.
   1. What happens to the oil price, initial oil extraction (R_0), and the "expected life time of oil reserves"?
   2. Are there any other things that may change, and how would that affect your initial analysis?
2. Suppose that instead of the usual assumption about competitive markets, a monopoly manages the resource.
   1. What happens to the resource price, initial extraction (R_0), and the "expected life time of the reserve/resource"?

      Comment: In this scenario one may run into some inconsistencies as one hits the choke price $K = P_T$ earlier that perod *T*, which could imply that the initial extraction is set incorrectly. A true cartel or monopolist would, if it knew what the exact amount of resources were, adjust extraction further to maxmize its profits (at the possible expense of lower welfare for society).
   2. Are there any other things that may change, and how would that affect your initial analysis?