ECN 275/375 Environmental and natural resource economics
12: Efficient & optimal use of natural resources
(Perman *et al.* Ch 14)

Reading guide

Session 11 dealt with the first half of Ch. 14. Focus on sec. 14.5 (the social welfare function in a dynamic context), and sec. 14.7 (complications). [just read sec. 14.6 – generalization to renewables – for a basic understanding].

Some concepts, notation, terminology, and math for dynamics

State variable – example: How many liters **are** in a tank

Flow variable – example: How many liters **flow** into a tank

	Continuous	Discrete		Continuous	Discrete
State variables	$S(t,...)$	$S_t(...)$	Time derivative:	$\dot{S}=\frac{\partial S(t,...)}{\partial t}$	$\Delta S_t(...)=S_t-S_{t-1}$
Flow variables:	$R(t,...)$	$R_t(...)$	Time derivatives usually not so essential, but same notation		

	Continuous time	Discrete time
Relation state-flow variable	$\dot{S}(t,...)=R(t,...)$	$S_t(...)=S_{t-1}(...)+R_t(...)$ OR $\Delta S_t(...)=S_t-S_{t-1}=R_t$

Remark: $R(t,...)>0\Rightarrow\dot{S}(t,...)>0$ OR $R(t,...)<0\Rightarrow\dot{S}(t,...)<0$ (continuous time)

$R_t(...)>0\Rightarrow\Delta S_t(...)>0$ OR $R_t(...)<0\Rightarrow\Delta S_t(...)<0$ (discrete time)

Social welfare over time

(see also sec. 3.5.1)

Basic idea: discounting = future income/utility worth less than current income/utility. As in utility theory the idea about *indifference* is essential.

$K_0=K_t\left(\frac{1}{1+\delta}\right)^t$ where δ (≥ 0) is the individual's discount rate (= impatience in consumption)

Let K_t denote the capital stock in time period t. To be indifferent between receiving K_t in time period t and K_0 in time period 0, capital must grow with the rate δ. Let r be the interest rate at which capital K grows. The complete mathematical expression for indifference.

$K_0=K_t\left(\frac{1}{1+\delta}\right)^t=K_0\left(\frac{1+r}{1+\delta}\right)^t$ (continuous time version: $K_0=K_t e^{-\delta t}=K_0 e^{(r-\delta)t}$)

Decision rule: $r>\delta\Rightarrow$ receiving capital in the future is preferred.
$r<\delta\Rightarrow$ receiving capital now is preferred

A generic welfare optimization problem for a non-renewable resource (eqs. 14.10-14.13)

Resource depletion: $\dot{S}_t = -R_t$ (14.10) [resource stock declines with yearly use]

Change in capital: $\dot{K}_t = Q_t - C_t$ (14.11) [capital change: production less consumption]

Production: $Q_t = Q_t(K_t, R_t)$ (14.12) [generic prod.fnc as before]

This gives the following dynamic social welfare maximization problem:

$$\begin{Bmatrix} MAX \\ C_t, K_t, R_t \end{Bmatrix} W = \begin{Bmatrix} MAX \\ C_t, K_t, R_t \end{Bmatrix} \int_{t=0}^{\infty} U(C_t) e^{-\rho t} dt$$ where ρ is the discount rate

subject to the following constraints

(1) Resource depletion $\dot{S}_t = -R_t$ (14.10) and

(2) Capital change $\dot{K}_t = Q_t(K_t, R_t) - C_t$ (combined 14.11 and 14.12)

which gives the following constrained maximization problem:

$$\mathcal{L} = [\int_{t=0}^{\infty} U(C_t) e^{-\rho t} dt] + P_t(-R_t) + \omega(Q_t(K_t, R_t) - C_t)$$

The solution to this problem gives the following equations

- $U_{c,t} = \omega_t$ (14.14.a) the marginal utility of consumption over time equals the shadow price ω_t on the consumption constraint over time)
- $P_t = \omega_t Q_{R,t}$ (14.14.b) the (shadow) price over time equals the shadow price price ω_t on the consumption constraint times the marginal product of the resource over time
- $\dot{P}_t = \rho P_t$ (14.14.c) the change in price over time equals the discount rate ρ times the price over time
- $\dot{\omega}_t = \rho \omega_t - Q_{K,t} \omega_t = \omega_t(\rho - Q_{K,t})$ (14.14.d) the change in the shadow price of consumption over time equals the discount rate times the shadow price of consumption over time less the marginal product of capital times the (OR the shadow price of consumption times the difference between the discount rate and the marginal product of capital over time)

Some remarks on the solution:

1. Due to discounting, higher consumption (C_t) and higher resource use (R_t) take place in the early time periods.
2. Capital (K_t) has no value except as an input in the production function (14.12 and 14.14.d)

Read section 14.5.2 for further interpretations of the optimal solution

Linkages to macro economics

The capital exchange equation $\dot{K}_t = Q_t(K_t, R_t) - C_t$ has parallels to the national accounts equation of gross domestic product (GDP) in macro: $Y = C + G + I$ (we exclude international trade as the resource issues can be viewed as global, and hence all trade is "domestic" = within the system boundaries. C and G are both consumption = C' which gives $Y = C + G + I = C' + I \Rightarrow I = Y - C'$.

Let: $Y = Q_t(K_t, R_t)$ and $\dot{K}_t = I$ (change in capital = net investment). We get:

$\dot{K}_t = Q_t(K_t, R_t) - C_t$

Extensions

This model can be expanded adding the costs of resource extraction. Let these costs be given by:

$\Gamma_t(R_t, S_t)$ with the following derivatives:

$\dfrac{\partial \Gamma(R_t, S_t)}{\partial R_t} > 0$ [costs increase with increasing resource extraction], and

$\dfrac{\partial \Gamma(R_t, S_t)}{\partial S_t} < 0$ [cost increase as the stock of the resource declines – a reasonable assumption as one would extract easily available resource stock first]

This addition affects only the capital change equation: $\dot{K}_t = Q_t(K_t, R_t) - C_t - \Gamma(R_t, S_t)$. Reason: including extraction costs reduces available capital over time.

Complications

Note the four points on p. 502 (where the three first relate to uncertainty of the quality of the resource stock, and the last relates to changes in extraction costs).

Hotelling's rule (for resource use)

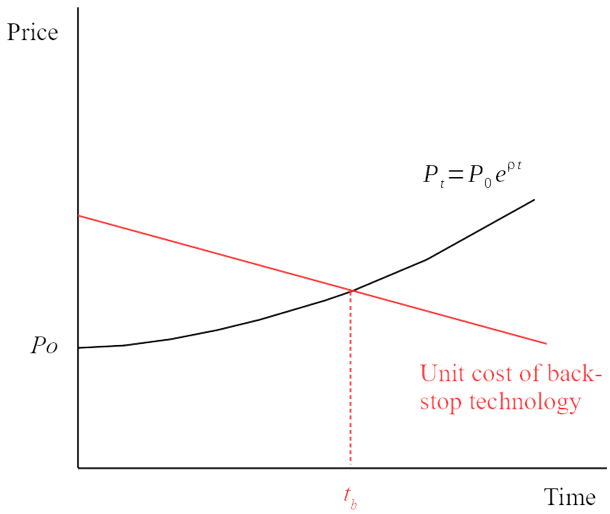
The long term net price (less extraction costs and transport costs) should equal the capitalized value of the initial resource price (at $t = 0$) over time for the owner of the resource to be unable to increase resource rents by changing the extraction profile.

Mathematically:

$P_t = P_0 e^{\rho t}$ for continuous time

$P_t = P_0 (1+\rho)^t$ for discrete time

Backstop technology = the technology that replaces (or reduces) resource use. Usually assumed to be declining over time. At time t_b the unit (marginal) costs of using the backstop technology equals the resource price, and the backstop technology takes over.



Exercises

See exercise tab on course page.

Discussion topics

None for this session – this is "new" material for many of you, so more time for questions and an early look (starting help) on the exercises.