ECN 275/375 Environmental and natural resource economics 12: Supplement – time derivative of a state variable

$\dot{S}_t = \delta S_t$: Intuitive explanation with δ as the interest rate – the growth equals initial capital stock S_t times the interest rate (think of S_t as an initial capital stock, and it grown by δ).

Continuous time:

Mathematically (conveniently for the time interval 0 to t):

$$S_t = S_0 e^{\delta t} \Rightarrow$$
$$\frac{\partial S_t}{\partial t} = \dot{S}_t = \delta S_0 e^{\delta t} \Rightarrow$$
$$\dot{S}_t = \delta S_t \, as \, S_t = S_0 e^{\delta t}$$

Discrete time:

The "time derivative in discrete time" is the growth between 2 time periods:

$$\Delta S_t = (1+\delta) S_t - S_{t-1}$$
$$= (1+\delta) S_{t-1} - S_{t-1}$$
$$= S_{t-1} + \delta S_{t-1} - S_{t-1}$$
$$= \delta S_{t-1}$$

Remark:

The difference between continuous time and discrete time, is that in discreate time the growth rate is applied to the previous time period, while in continuous time and with the recollection that derivatives relate to infinite small changes, we get the continuous time result as a function of current time, t.