10: Game theory & cooperation The Folk theorem & side payments

- Objectives
  - show how non-cooperative single shot games can yield cooperative outcomes when they are made dynamic = demonstrate the Folk theorem
  - side-payments as a vehicle for cooperation

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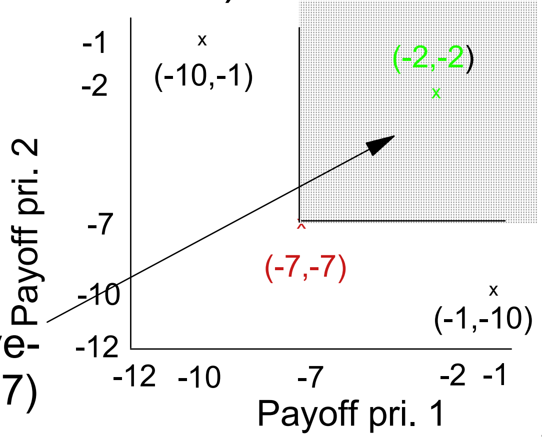
Nash equilibrium - repetition (1)

- Definition Nash equilibrium: The outcome that results when a player plays his/her **best reply strategy** given that all the other players play their best reply strategy
- Problem: Nash equilibria are rarely Pareto-optimal (in that sense a pessimistic outcome)

	Prisoner 1	
Prisoner 2:	Don't accuse	Accuse
Don't accuse	(-2,-2)	(-1,-10)
Accuse	(-10,-1)	(-7,-7)



Region of potential Pareto improve-ment from non-coop solution (-7,-7)

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The Folk theorem (1)

- Demonstrates how cooperative outcomes (that differ from the single shot Nash equilibrum) may occur in noncooperative settings
- Requirement: infinitely repated games
  - ... or a game with random stop time [has same effect as infinite stop time as backwards recursion then is not applicable]
- Definition of the Folk theorem
  Any individually rational pay-off vector can be supported as a Nash equilibrium in repeated games that last forever and the discount rate is sufficiently low.

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... the Folk theorem (2)

Dynamic concept - main intuition

$NPV_{nice} \geq NPV_{bad}$ for all agents in the game

nice bad

$$\sum_{t=o}^{\infty} \beta_i^t \pi_{i,t}^c \geq \left(\sum_{t=o}^{T-t} \beta_i^t \pi_{i,t}^c\right) + \beta_i^T \varphi_{i,T} + \left(\sum_{t=T+1}^{\infty} \beta_i^t \pi_{i,t}^n\right) \quad [1]$$

β_i the discount factor, $\frac{1}{1+r_i}$, for agent *i*

$\pi_{i,t}^c$ the payoff to agent *i* of playing cooperatively in period *t*

$\varphi_{i,t}$ the best reply strategy to agent *i* given that the other players play cooperatively in period *t*

$\pi_{i,t}^n$ the payoff to agent *i* when all agents pay non-coop

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... the Folk theorem (3)

Solving [1] is complicated (non-linear). [1] can be divided into a series of 2-period games, and each 2-period game needs to satisfy the

$NPV_{nice} > NPV_{bad}$ criterion

Reducing [1] to a 2-period sub-game:

$$\sum_{t=o}^{1} \beta_i^t \; \pi_{i,t}^c (= \beta^0 \pi_{i,0}^c + \beta^1 \pi_{i,1}^c) \geq \beta_i^0 \; \varphi_{i,0} + \beta_i^1 \; \pi_{i,1}^n = \; \varphi_{i,0} + \beta_i \; \pi_{i,1}^n \quad [2]$$

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... the Folk theorem (4)

The solution to [2] in a setting where $t = 0$ and $t + 1 = 1$:

$$1 > \beta_i \geq \frac{\varphi_{i,0} - \pi_{i,0}^c}{\pi_{i,1}^c - \pi_{i,1}^n} \quad \forall i \in I \quad [3]$$

The general format, where t can take on any value within the unknown timeframe of the game, T

$$1 > \beta_i \geq \frac{\varphi_{i,t} - \pi_{i,t}^c}{\pi_{i,t+1}^c - \pi_{i,t+1}^n} \quad \forall \; i \in I, \forall \; t \in T \quad [3']$$

If [3] (or [3']) holds for all agents, it is in all the agents' best self interest to play "nice"

i.e., a cooperative outcome in a non-cooperative setting is achieved

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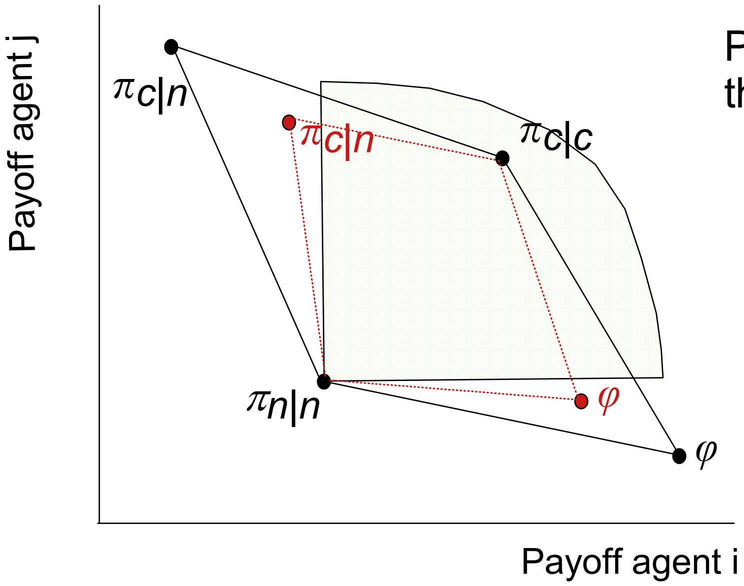
... the Folk theorem (5)

Graphical representation (from agent i's perspective)



Profit ranking for the Folk theorem to make sense:

$\varphi > \pi_{c|c} > \pi_{n|n} > \pi_{c|n}$

The less spread out in NW-SE directions, the more likely it is that the Folk theorem holds (cfr. [3])

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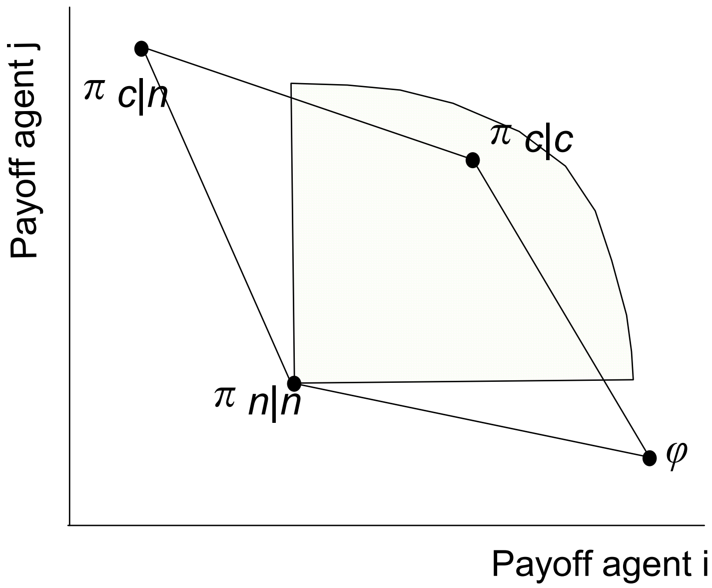
The safety level in a game

- Best payoff that a player is se- cured without relying on co- operation from other players = safety level
- Here (for both players): $\pi_{n|n}$
- Rule: no agent accepts a pay- off below his security level



(from agent i's perspective)

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The negotiation space (1)

- Folk theorem welfare ranking: $\varphi > \pi_{c|c} > \pi_{n|n} > \pi_{c|n}$: source to many of the difficulties in reaching welfare enhancing cooperative outcomes
- No general rule for obtaining cooperative outcomes in "nonrepeated" games
- ... but side-payments/-penalties move the game to situations where solution is "trivial"
- Some key difficulties using payments/ penalties
  - they reduce the payoff to the players who start using them (offers a side-payment)
  - often "a last mover advantage" (= benefits to wait for other players to offer side-payment)

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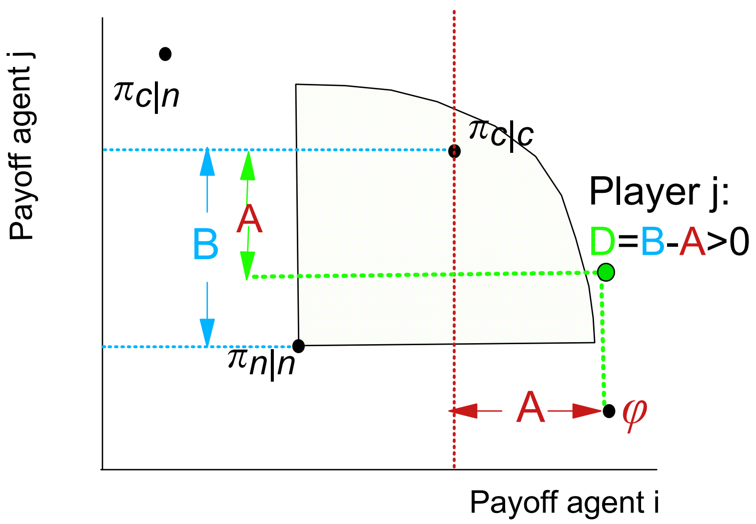
.. the negotiation space (2)

- A : compensation needed for player *i* not to play φ if *j* plays coop.
- B : max payment *j* can offer *i* to sign agreement for *j* to be equally well off
- D : ex-payment payoffs



Problem: who moves first (= offers a side-payment)?

Here: *j* moves first ==> reduces gain from cooperation by size of side-payment A to induce *i* to cooperate

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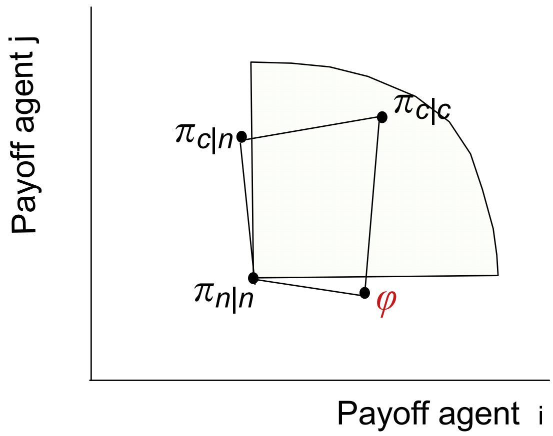
.. the negotiation space (3)

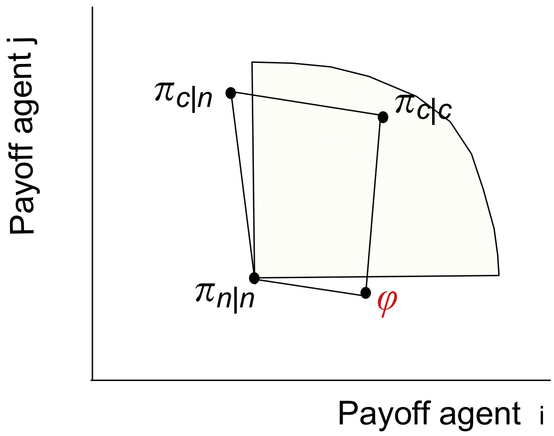
Noncooperative games with "trivial" solutions
= breaking the Nash payoff ranking curse

$\varphi > \pi_{c|c} > \pi_{n|n} > \pi_{c|n}$

Assurance game - solution given



Mixed game - side payment
+ given sequence of moves



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Concluding remarks

- Cooperative outcomes can be achieved in repeated non-cooperative games through the Folk theorem
  - applicable to a special class of repeated games
  - random stop time
  - payoff difference between the best reply strategy (Nash setting) and cooperation is not too large
  - the discount rate is not too large
  - adaptable ==> adjust game formulation
- Side-payments
  - no specific rules - main issue: who moves first?
  - first-mover disadvantage: reduces own payoff to compensate other players

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