A simple market game
(to illustrate the relevance of strategic actions)

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There are 5 firms in a potential pollution permit market. Only the firm itself knows its own marginal abatement cost function, $MAC_i\,(m_i)$. Each firm's initial emission level is m_i^o, and each firm is freely given pollution permits that is half of its initial emission level, i.e., $m_i = \frac{1}{2} m_i^o$. Before the tradable permit market was introduced, total emissions, $M^o = \Sigma_i\, m_i^o = 434$. The benefits from emissions reductions are not known, but the politicians have firmly decided to cut emissions to half of the initial amount. The initial marginal abatement cost functions are all of the format $MAC(m_i) = (m_i^o - m_i) / \sqrt{m_i^o}$.

Purpose of exercise: Show that because (at least some of) the firms have different marginal abatement cost functions, there are potential gains to be made from trade. This will be manifested by trade starting despite no starting price from the regulator.

Extra: By making an investment of $m_i^o \sqrt{m_i^o}$ firms can get the new marginal abatement cost function $MAC_{new}(m_i) = ½\,(m_i^o - m_i) / \sqrt{m_i^o}$. Firms can borrow at the market interest rate r'. Is this investment profitable? Explain why it is more profitable for some firms than others. [this is not part is not to be done in the standard class room exercise]

Suggested answer

This brief summary on how to approach the problem only deals with the trade part of the question, not the issue of investment.

The key issue for each firm is to figure out what their marginal abatement costs are for various emission levels, and prior to prices being posted on the exchange, if they should post an offer to sell or buy permits.

The optimal emission level z_i can be expressed as a function of the initial (pre-trade) emission level plus the bought volume (or minus the sold volume). With the allocated emission levels being halved, the emission level expressed in terms of the traded volume Δz_i becomes $z_i = \frac{1}{2}z_i^o - \Delta z_i$. Inserting this into the original *MAC* function $MAC_i(z_i) = \frac{z_i^o - z_i}{\sqrt{z_i^o}}$ yields :

$$MAC_i(\Delta z_i) = \frac{z_i^o - (\frac{1}{2}z_i^o - \Delta z_i)}{\sqrt{z_i^o}} = \frac{\frac{1}{2}z_i^o + \Delta z_i}{\sqrt{z_i^o}} \qquad [1]$$

Prior to any trades having occurred, $\Delta z_i = 0$. To figure out what one should offer to sell or bid to buy permits (or what posted selling and buying prices to accept) one has to compare [1] with the mean *MAC* for the rest of the market, evaluated at an emission level that is half of the initial emission level. The point estimate of the *MAC* for the rest of the economy is total emissions minus own emissions, i.e.,

$$MAC_{rest}(z_{rest}) = \frac{\frac{1}{N-1}((Z^o - z_i^o) - \frac{1}{2}(Z^o - z_i^o - \Delta z_{rest}))}{\sqrt{\frac{Z^o - z_i^o}{N-1}}} = \frac{\frac{1}{2}(Z^o - z_i^o) + \frac{\Delta z_{rest}}{N-1}}{\sqrt{N-1}\,\sqrt{Z^o - z_i^o}} \qquad [2]$$

where Z^o denotes aggregate emissions (= 608), *N* denotes total number of firms (= 7), and z_i^o denotes firm i's emissions. Note that Z^o and *N* is common information, while z_i^o is private information, i.e., only firm *i* knows this.

Given the common information in this game (the number of players, the total emission level, and the type of the cost function) it is possible for any firm to calculate if it is a potential buyer or a seller. This is done by comparing their own *MAC* [1] with the *MAC* of the mean firm in the rest of the economy [2].

Evaluating [1] at the pre trade level, i.e., Δz_i^o is zero, yields column two in the table below., while the third column is obtained from [2]. The fourth column offers a recommended starting strategy.

Firm i's initial emission level (z_i^o)	Firm i's pre-trade MAC (from [1])	Rest of firms' mean pre-trade MAC (from [2])	Advice to firm i before trading starts
64	4.00	4.76	offer to sell one unit so that $p_{offer} > 4.76$
144	6.00	4.40	bid to buy one unit at $p_{bid} < 4.40$

Note that for the large firm (z_i^o = 144) the *MAC* (144) = 6 is much higher than the rest of the firms' mean pre-trade *MAC*, indicating that the large firm most likely is a buyer. Similarly, the small firm (z_i^o = 64) is most likely a seller. Note that firms in the initial round chooses to offer to sell or buy, their asking prices exceed their own *MAC*s evaluated at z_i^o. The reason for this is that in the initial rounds of trading firm managers seek to learn about the cost schedules of the other firms given their observed behavior in the market, i.e., at what prices the other firms choose to buy or sell permits.

In optimum we know that $p = MAC_i(\bar{z}_i + \Delta z_i^*) = MAC_j(\bar{z}_j + \Delta z_j^*) \;\; \forall i,j \in I$, where $\bar{z}_i$ and $\bar{z}_j$ denote the pre-trade allocation, and Δz_i^* and Δz_j^* denote the optimal amount to trade for firms *i* and *j*. Vis-a-vis the rest of the economy any firm can therefore do better than the table above by solving for the trade volume they should have for the equimarginal principle to hold. This implies that firm i's traded (bought or sold) volume, Δz_iequals the negative of the net traded volume in the rest of the economy. In optimum (all arbitrage possibilities exploited)

$$MAC_i(\bar{z}_i + \Delta z_i) = MAC_{rest}(\bar{z}_{rest} - \tfrac{\Delta z_i}{N-1}) \qquad [3]$$

where $\bar{z}_{rest}$ is the pre-trade level for the mean rest firm in the economy. Rewriting [3] using [1] and [2] yields:

$$\frac{\frac{1}{2}z_i^o + \Delta z_i}{\sqrt{z_i^o}} = \frac{\frac{1}{2}(Z^o - z_i^o) - \Delta z_i}{\sqrt{N-1}\,\sqrt{Z^o - z_i^o}} \qquad [4]$$

Note that if Δz_i is positive for firm *i*, it must be negative for the mean net trade of the other firms. Also note that the mean rest firm trades a quantity equaling $\frac{-1}{N-1}$ of firm *i*'s traded quantity as there are N-1 of these firms, and if *i* sells, the rest buys, and conversely . Solving for Δz_i yields the following expression after some transformations:

$$\Delta z_i = \frac{\sqrt{z_i^o}\,(Z^0 - z_i^o) - z_i^o\sqrt{N-1}\,\sqrt{Z^0 - z_i^o}}{-2\left(\sqrt{N-1}\,\sqrt{Z^0 - z_i^o} - \sqrt{z_i^o}\right)} \qquad [5]$$

For a firm with the initial emission level, $z_i^0 = 64$, using [5] Δz_i becomes:

$$\Delta z_i = \frac{\sqrt{64}(608-64)-64\sqrt{7-1}\sqrt{(608-64)}}{-2(\sqrt{7-1}\sqrt{608-64}+\sqrt{64})} = -5.34 \qquad [5']$$

i.e., it should sell. Now consider the firm with the initial emission level, $z_i^0 = 144$. [5] then becomes:

$$\Delta z_i = \frac{\sqrt{144}(608-144)-144\sqrt{7-1}\sqrt{(608-144)}}{-2(\sqrt{7-1}\sqrt{608-144}+\sqrt{144})} = 15.67 \qquad [5'']$$

i.e., it should buy. How much the firms should sell or buy depends on the distribution of the MACs in the rest of the economy, but [5'] and [5''] provide an indication.

Comparing these results with a more complete information setting, where the number of firms in each category is known yield some interesting insights. Solving for the incomplete information setting involves finding an equilibrium price that clears the market. Market clearing implies that quantities sold equal quantities bought. In this example, there are only two types of firms, 5 firms with $z_i^o = 64$, and 2 firms with $z_i^0 = 144$.[1]

With two types of firms we get the following equations to solve:

- price clears : $MAC_{i,sell} = (\frac{\frac{1}{2}z_{i,sell}^o + \Delta z_{i,sell}}{\sqrt{z_{i,sell}^o}}) = MAC_{i,buy} (= \frac{\frac{1}{2}z_{i,buy}^o + \Delta z_{i,buy}}{\sqrt{z_{i,buy}^o}})$ [6a]

- quantity clears: $5\Delta z_{i,sell} + 2\Delta z_{i,buy} = 0 \Rightarrow \Delta z_{i,sell} = -\frac{2}{5}\Delta z_{i,buy}$ [6b]

Inserting [6b] into [6a] yields:

$$\frac{\frac{1}{2}z_{i,sell}^o - \frac{2}{5}\Delta z_{i,buy}}{\sqrt{z_{i,sell}^o}} = \frac{\frac{1}{2}z_{i,buy}^o + \Delta z_{i,buy}}{\sqrt{z_{i,buy}^o}} \qquad [7]$$

which after some transformations gives:

$$\Delta z_{i,buy} = \frac{\frac{1}{2}\left(z_{i,buy}^o\sqrt{z_{i,sell}^o} - z_{i,sell}^o\sqrt{z_{i,buy}^o}\right)}{-\frac{2}{5}\sqrt{z_{i,buy}^o} - \sqrt{z_{i,sell}^o}} = \frac{\frac{1}{2}(144\sqrt{64} - 64\sqrt{144})}{-\frac{2}{5}\sqrt{144} - \sqrt{64}} = 15 \qquad [8]$$

i.e., each buying firm should buy 15 units in optimum, implying that each selling firm sells 6 units. This yields the equilibrium price $4\frac{2}{3}$by inserting traded volumes into [1] for either the large or small firm (You may check the correctness of the calculations by inserting the traded volume for the other type of firm - you should get the same equilibrium price. Why?)

[1] That makes calculating the tradable permit market equilibrium more complicated as we get a larger number of equations that needs to be solved simultaneously.

There some small deviation between [5'] and [5"] on one side, and the results derived from [8] on the other hand. This is because operating with a mean in the rest of the market, the implied heterogeneity in the market becomes less. However, for each firm type, the sign of the action (to sell or buy) is correct.

The difference in the transition from the incomplete information to the complete information setting, underlines the rationale for starting trading rather carefully. As prices are revealed, agents learn more about the other agents. In brief, to try to extract some rents, one should therefore offer to sell (if one is a potential seller) at a higher rate than this mean, and to buy (if one is a potential buyer) at a lower rate than this mean.