## ECN 275/375 Environmental and natural resource economics
## Supplement lecture 2: Single input production

Consider production of wheat. Total yields are given as a function of the amount of nitrogen applied. Timing of applying the Nitrogen and other factors are left out. Produced quantity per hectare, $Y$, is then a function that can be written as $Y = f(N)$, where $N$ is the number of kilograms of Nitrogen applied per hectare. The function $f(N)$ is continuous and twice differentiable, i.e., the first and second order derivatives exist for the entire domain of the function.

Grain prices are $p$ per kilogram, and nitrogen costs are $v$ per kilogram. All prices are exogenous, i.e., beyond the farmer's control, and positive. Fixed costs are denoted $FC$. The amount of Nitrogen to apply, $N$, is therefore the only choice variable in this model. Per hectare profits, $\pi$, can then be written as follows:

$$\pi(N) = TR - TC = pY - vN - FC = pf(N) - vN - FC \qquad [1]$$

To find the profit maximizing quantity of nitrogen, differentiate this function with respect to nitrogen to get the first and second order conditions **for profit maximizing fertilization**:

FOC: $\pi'(N) = pf'(N) - v = 0$ which is rearranged to become $f'(N) = \frac{v}{p}$ [2]

SOC: $\pi''(N) = pf''(N) < 0$ which is simplified to $f''(N) < 0$ as $p$ is positive [3]

Figure 1 illustrates the total and marginal perspectives.

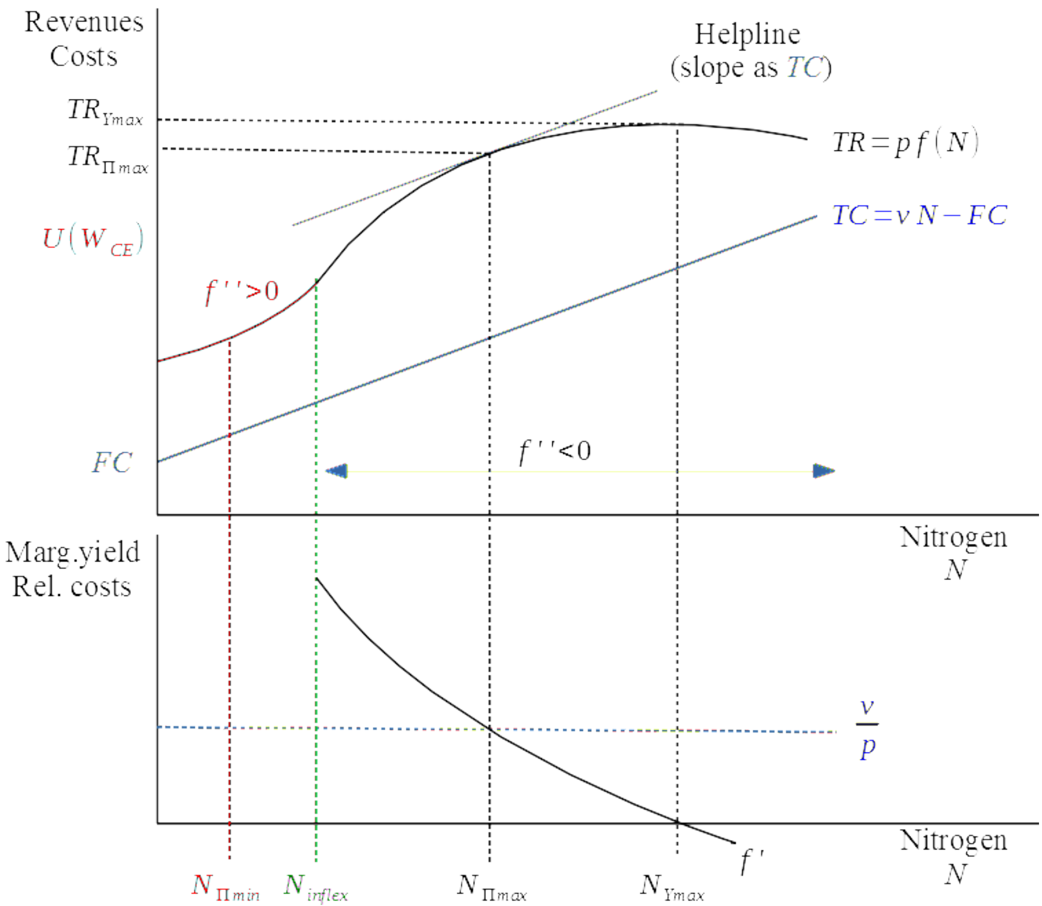


**Figure 1:** Production economic perspectives of N-fertilization.

The top panel of figure 1 shows equation [1] divided into total revenues (*TR*) and total costs (*TC*), while the lower panel shows equation [2] with the marginal physical product (*f'*) and the relative prices (*v/p*). The profit maximizing quantity of nitrogen is $N_{\Pi max}$, where the marginal physical product crosses the relative price line (*f'* = *v/p*) in the lower pane.. This is the fertilization level where the distance between total revenues and total costs are the greatest in the top panel. Note that at the fertilization level $N_{\Pi min}$, profits are the lowest except for extremely high fertilization levels beyond $N_{Y max}$, the maximum yield fertilization. Also note that $N_{Y max}$ can only be the profit maximizing fertilization level if the price of Nitrogen, *v*, is zero.

We are now going to look at what happens to fertilization levels when the product price or the fertilizer price increases. Consider a product price increase, *s* > 0, first. Rewriting equation [2], we get:

$$f'(N)=\frac{v}{p+s}<\frac{v}{p} \qquad [3]$$

This implies that the relative price line *v/p* in the bottom panel drops, and it becomes profitable to increase the fertilization level. Note that the price increase gives a rather modest increase in yields. Suppose Nitrate leaching is likely to increase with fertilization beyond $N_{inlex}$. Hence, a rather modest increase in total yields comes at a cost to the environment through higher eutrophication which hardly can be claimed to be sustainable if we are concerned about water quality.

Now consider an increase in the fertilizer price *t* > 0. Rewriting equation [2], we now get:

$$f'(N)=\frac{v+t}{p}>\frac{v}{p} \qquad [4]$$

This implies that the relative price line in the bottom panel of Figure 1 moves up, and that the profit maximizing fertilization level declines. Consistent with the exposition for the price increase, lower fertilization rates to the right of $N_{inlex}$ should lower Nitrate leaching, but at a much larger cost due to the curvature of the total revenue curve.