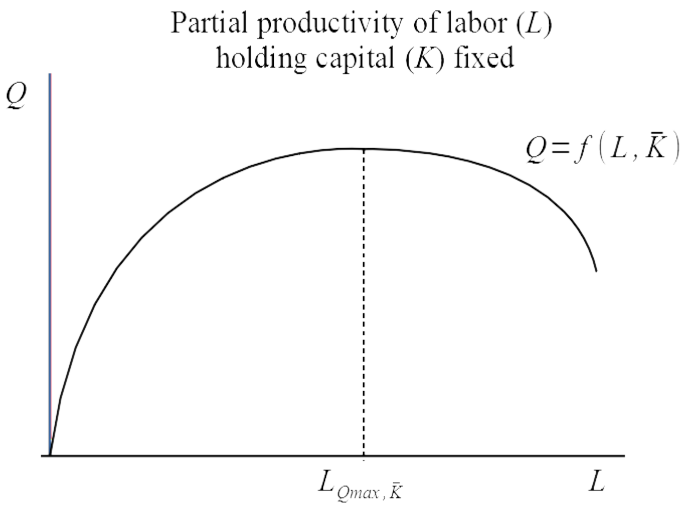
ECN 275/375 Environmental and natural resource economics
Supplement lecture 2: Multi input production

Consider a production function for a single product *(Q)* with two inputs, labor (*L*) and capital (*K*) of the following form:



$Q=f(L,K)$ with positive but declining marginal products for labor

1. partial der.: $\frac{\partial Q}{\partial L}=f_L(L,K)>0\ \forall\ L<L_{Qmax,\bar{K}}$, and
2. partial der.: $\frac{\partial^2 Q}{\partial L^2}=f_{LL}(L,K)<0$

Similarly for capital with positive but declining marginal products:

1. partial der.: $\frac{\partial Q}{\partial K}=f_K(L,K)>0\ \forall\ K<K_{Qmax,\bar{L}}$, and
2. partial der.: $\frac{\partial^2 Q}{\partial K^2}=f_{KK}(L,K)<0$

With *p as* the product price, and *w* (wages) and *r* (interest rate) as the respective costs of labor and capital, the profit function for this production is now given by:

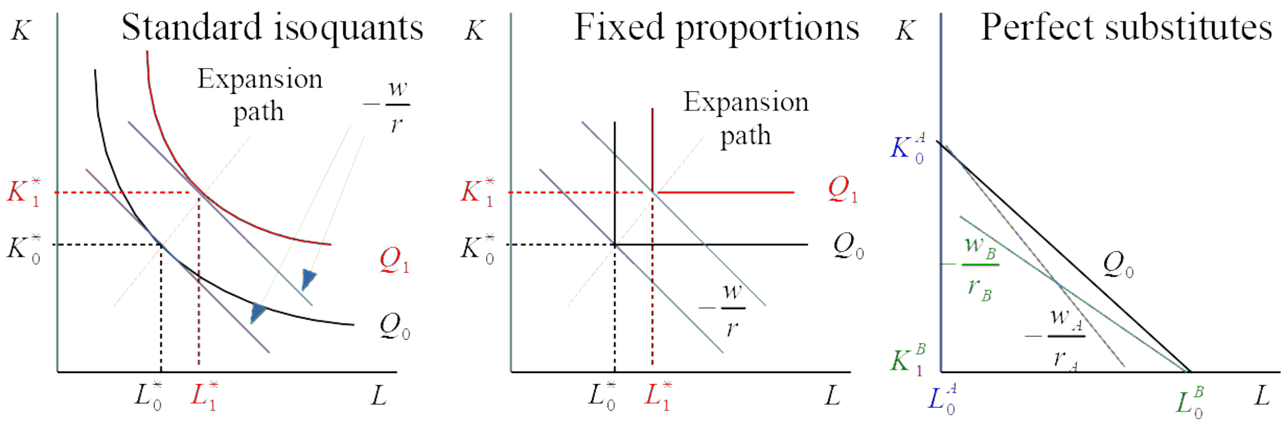
$$\pi=pQ-wL-rK=pf(L,K)-wL-rK$$

with the following first order conditions for profit maximization:

$$\frac{\partial \pi}{\partial L}=pf_L(L,K)-w=0\text{, and }\frac{\partial \pi}{\partial K}=pf_K(L,K)-r=0$$

To better understand the properties of this production function, consider the isoquants that are derived from the combination of labor and capital that gives a given quantity, Q_0, (or Q_0) of the product. We gain economic intuition by structuring this as a quantity restricted cost minimization problem:

$\min\limits_{(L,K)} wL+rK$ subject to $f(L,K)\geq Q_0$, which is solved using the Lagrange method.



(1) Standard isoquants: Optimal mix of inputs (*L* and *K*) changes when relative prices ((*w* and *r*) change. An increase in produced levels are driven by increases in the product price.

(2) Fixed proportions: Optimal input mix does not change as relative prices change.

(3) Perfect substitutes: Corner solutions optimal, i.e., (all of one factor, none of the other) depending on slope of the relative input price line being steeper or flatter than the isoquant.