ECN 275/375: Sustainability

(EX5-1) The Brundtland commission

The Brundtland commission's definition of sustainable development:

> *... development that meets the **needs of the present** without compromising the ability of future generations to meet their own needs ...*

(a) Propose a definition of needs (of the present generation and for future generations) that you can implement in a model.

Answer: My answers are as good (or bad) as yours. The challenge is to find formulations that do not unnecessarily constrain solutions with the result that welfare falls more than needed to meet the objective in the above quote. Remember: constraints provide the setting in which optimization can take place.

(b) One of the aims of economics is to maximize social welfare, which then leads to a utility function formulation. With increasing population numbers a per capita specification of utility may be warranted. Discuss the following objective function for a representative world citizen:

$U(C_t, E(R_t))$

where C_t is **consumption per capita** with partial derivative $U_C(C_t)>0$,

$E(R_t)$ is an externality indicator where R_t is resource use with partial derivative

where $U_E(C_t, E(R_t))<0$ and $E_R(R_t)>0$

Does this utility function capture the essentials for analyzing sustainability, or are some key elements left out? If so, which key elements are left out? Explain briefly.

Answer: The specified objective function misses out on accumulated pollution (for climate gas emissions it is the accumulated impacts that are of greatest concern). A revised version of the utility function is $U(C_t, E(R_t, A_t))$ where A_t captures the missing external impact of accumulated emissions. This gives the following changes to the externality indicator description: $U_E(C_t, E(R_t, A_t))<0$, $E_R(R_t, A_t)>0$, and $E_A(R_t, A_t)>0$. The revised utility function (addition in red) then becomes:

$U(C_t, E(R_t, A_t))$

I would also suggest to add declining marginal utility of consumption: $U_{CC}(C_t)<0$ as that helps to capture intergenerational equity. One may discuss if this also helps solve some intragenerational equity issues.

When structuring a model, it is often easier to formulate it in discrete than continuous time. To see this, consider the following state variable: $S_t=(1+g)S_{t-1}$ where g is the growth rate of the state variable. The equivalent continuous time formulation is $\dot{S}_t=g S_t$, which expressed in discrete time becomes $S_t - S_{t-1} = g S_{t-1}$. Adding S_{t-1} to both sides of this equation yields the discrete time formulation. Something to think about when formulating the various parts of your model. Note that in continuous time, $\lim_{\Delta t \to 0} S_t = S_{t-\Delta t}$.

(c) Population growth has been exponential for some time, but it is expected to taper off such that population numbers are expected to stabilize around 2050. How to include population number changes in your model?

Answer: $N_t = (1+\gamma_t) N_{t-1}$ where N_t is population number in period t and a time varying populatioin growth coefficient γ_t captures time varying changes in population numbers. As the capital change constraint is in continuous time, it is analytically clearer to use the continuous time version of the demographics equation, i.e., $N_t = N_0 e^{\gamma t}$. Remark: We could make the exponent more complicated to allow for the decline in x over time, but there is a value in keeping the model as simple as possible and still capture the essentials.

My suggestion is to add $N_0 e^{\gamma t}$ to the capital change constraint as C_t: is per-capital consumption given the formulated question.

$\dot{K} = Q(K_t, R_t, E(R_t, A_t)) - N_0 e^{\gamma t} C_t - \Gamma(R_t) - V_t$

Next, I would split my model in two parts, with one part covering the period with population growth until T, i.e., $\int_0^T U(C_t) e^{-rt} dt$ and for the situation with zero population growth $\int_T^\infty U(C_t) e^{-rt} dt$. For $t > T$, the above capital change constraint would simplify to $\dot{K} = Q(K_t, R_t, E(R_t, A_t)) - N_T C_t - \Gamma(R_t) - V_t$ as $N_T = N_0 e^{\gamma T}$.

(d) How would you formulate the resource constraints?

Answer: The objective function does not specify any difference between non-renewables R_t and renewables. If we are only concerned about non-renewables, the resource constraint can be written as $\bar{R} \geq \sum_{t=0}^{T} R_t$ with the auxiliary condition $\bar{R} \geq 0$.

If renewables (biological resources, B_t) do not cause any externalities, i.e., the externality indicator remains the same $E(R_t, A_t)$, and we only need to add a renewable constraint. One way to write this constraint is $B_t = B_{t-1} + \beta(B_{t-1}) B_{t-1} - H_t = (1+\beta(B_{t-1})) B_{t-1} - H_t$ where is the net growth and H_t is harvest in period t. We'll discuss renewables later in the course, and leave those out for now.

(e) How would you formulate the pollution constraints?

Answer: The immediate effects of pollution from resource use are already captured in the emission indicator, $E(R_t, A_t)$, while we need to model the change in accumulated emissions as we did in lecture 14: $\dot{A} = M(R_t) - \alpha A_t - F(V_t)$ where $M(R_t)$ are emissions due to resource use R_t, α is a self-cleaning factor, and $F(V_t)$ are cleaning efforts following funds, v, allocated for that purpose. In discrete time: $A_t = M(R_t) + (1-\alpha) A_{t-1} - F(V_t)$

CO_2-concentrations in the atmosphere constitute a major concern. We may therefore add an additional constraint here: $A_t \leq \bar{A}\ \forall t \geq 0$ to reflect this. If we allow accumulated emissions to be above the target $\bar{A}$ for some time, and we have a finite time horizon T, an alternative and less restrictive specification is $A_T \leq \bar{A}$.

(f) In lecture 14 the production function in continuous time is $Q(R_t, K_t, E(R_t, A_t))$ where man made capital, K_t, is the only undefined term in this exposition. How would you formulate the change in the stock of production capital?

Answer: The continuous time expression in lect. 14 $\dot{K} = Q(K_t, R_t, E(R_t, A_t)) - C_t - \Gamma(R_t) - V_t$ captures the essentials with one exception: we have formulated consumption C_t on a per capita basis. Therefore we need to add N_t before consumption as we have defined consumption C_t per capita. $\Gamma(R_t)$ are resource extraction costs. In discrete time:

$K_t - K_{t-1} = Q(K_{t-1}, R_t, E(R_t, A_t)) - N_t C_t - \Gamma(R_t) - V_t$. The time change on capital in the production function from K_t to K_{t-1} reflects that production takes place on capital available in the previous period. This is a "matter of taste" – just be aware of what you do in discrete time.

(g) Given that population growth is assumed to be zero some time around 2050, and climate issues need to be resolved also some time around 2050, consider formulating the objective function with the appropriate choice variables for a finite time horizon *T*.

Answer: $\begin{Bmatrix} MAX \\ C_t, R_t, V_t \end{Bmatrix} W = \begin{Bmatrix} MAX \\ C_t, R_t, V_t \end{Bmatrix} \sum_0^T U(C_t, E(R_t, A_t)) \left(\frac{1}{1+r}\right)^t$ (almost as in Lecture 14, but with a change in the upper time limit for the integral from infinity to *T*).

Note: This time horizon change may lead to an adjustment of the auxiliary condition in (d) to $\bar{R} \geq R_T$, where R_T are resources deemed necessary for sustained welfare beyond *T*.