ECN 275/375: A basic resource extraction model

(EX4-1) A basic static resource extraction model

Note: only one input in the production, the resource *R*

(a) Set up a basic profit maximizing model for unconstrained extraction of a natural resource *R* when extraction costs increase as extraction rates grow with higher extraction rates (an example of such a cost function would be California gold miners in 1849 – as your mining tunnel gets longer, mining becomes more time consuming).

(b) What are the necessary conditions for profit maximization? Explain.

(c) Let R^* be the profit maximizing level of resource extraction. Show that resource extraction declines if a resource extraction tax is added, i.e., $R^{tax} < R^*$.

(d) Let $\bar{R}$ denote the total amount of the resource. Why is it useful to formulate the constrained profit maximization problem with a "less than equal" rather than "equality" resource extraction constraint?

Remark 1: Let R^* be the solution of the unconstrained optimization problem, and let $\bar{R}$ be the required resource extraction (mining of *R*). Interpretation of λ (the constraint multiplier) in an "equality constrained" profit maximization problem:

1. $\lambda > 0 \Rightarrow R^* > \bar{R}$: The constraint reduces profits as the constraint "bites" (reduces the extraction of *R*)
2. $\lambda = 0 \Rightarrow R^* = \bar{R}$: The constraint (accidentally) coincides with the profit maximizing extraction level *R'*. Profits not affected by the constraint.
3. $\lambda < 0 \Rightarrow R^* < \bar{R}$: Forced extraction above the profit maximizing extraction level, *R'*. This also reduces profits. This take place if $MR(\bar{R}) - MC(\bar{R}) < 0$.

Remark 2: The equality constraint coincides with ordinary Lagrangian optimization. While I do not require you to know Kuhn-Tucker optimization for test 2, it is useful to know what it provides in terms of increased flexibility and what that implies:

1. The ≤ constraint is non-binding → the shadow price of the constraint (the equivalent of the Lagrangian multiplier) is zero.
2. The "≤ constraint" is binding → the constraint becomes an "equality constraint" which transfers the problem to a standard Lagrangian optimization problem, with the ordinary interpretation of the shadow price of the "equality constraint".

Question 2 on dynamic optimization of the same problem on the next page.

(EX4-2) A basic dynamic resource extraction model

Assume to technological progress, i.e., the production and cost functions do not change over time. This enables me to drop time subscripts in the production and cost functions to respectively write $Q(R_t)$ and $c(R_t)$. Note that production and costs may still vary over time,

My setup of the dynamic model based on my suggested solution of (1) then becomes:

$$\begin{Bmatrix} MAX \\ R_t \end{Bmatrix} NPV(R_t) = \begin{Bmatrix} MAX \\ R_t \end{Bmatrix} \int_0^\infty \pi_t(R_t) e^{-rt} dt = \begin{Bmatrix} MAX \\ R_t \end{Bmatrix} \int_0^\infty (p_t Q(R_t) - c(R_t)) e^{-rt} dt$$

(a) Explain why is the profit maximizing quantity to harvest R^* can be given an expression that resembles the static optima solution when prices are constant ($p = p_t \forall t$), i.e., we get the FOC $\frac{\partial \pi_t}{\partial R_t} = p Q_R(R_t) - C_R(R_t) = 0$.

(b) Formulate a constraint where the accumulated extraction of resources is less than or equal to the total availability of resources.

(c) Intuitively, why would the addition of this constraint affect the solution in (a)?