ECN 275/375: A basic resource extraction model

(EX4-1) A basic static resource extraction model

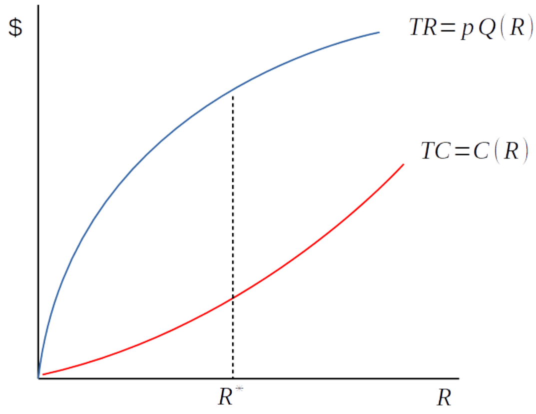
Note: only one input in the production, the resource *R*

(a) Set up a basic profit maximizing model for unconstrained extraction of a natural resource *R* when extraction costs increase as extraction rates grow with higher extraction rates (an example of such a cost function would be California gold miners in 1849 – as your mining tunnel gets longer, mining becomes more time consuming).

Answer: Profits equal total revenues (price x quantity) less mining costs (the cost function):

$$\pi(R) = TR(R) - TC(R)$$
$$= pQ(R) - C(R)$$

Note that $C(R)$ is a cost function in *R* with $C_R(R)>0$ and $C_{RR}(R)>0$ (increasing marginal costs). The production function has the followng properties: $Q_R(R)>0$ and $Q_{RR}(R)<0$ (decreasing marginal product). Profit maximizing quantity of *R* appears to be R^* .



Alternatively, a simpler model $\pi(R) = pR - TC(R)$ can be used. In that case the graphics (above, and below in (b)) simplify accordingly) without changing the essence of the analysis.

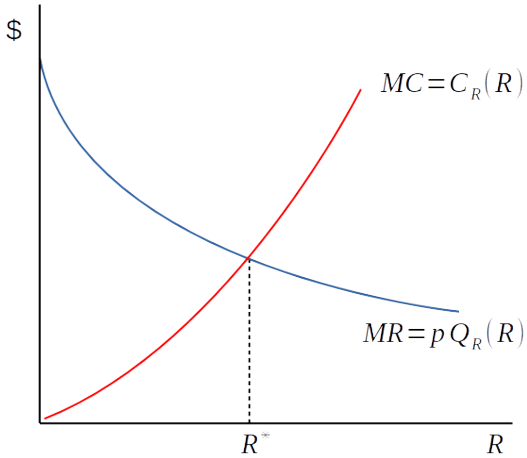
(b) What are the necessary conditions for profit maximization? Explain.

Answer: Differentiate the profit function with respect to the only choice variable, *R*, to get the first order condition (FOC) for profit maximization:

FOC: $\frac{\partial \pi(R)}{\partial R} = pQ_R(R) - C_R(R) = 0$

Note that with the properties of the production and cost functions in (a), R^* is a global maximum (no other level of *R* gives higher profits) for this problem. Also note that with:

- a concave production function it would suffice that the cost function was linear (like *v R*), or
- a convex cost function it would suffice that the production function was linear (like the mined gold is what is sold at the price *p* without any further processing)

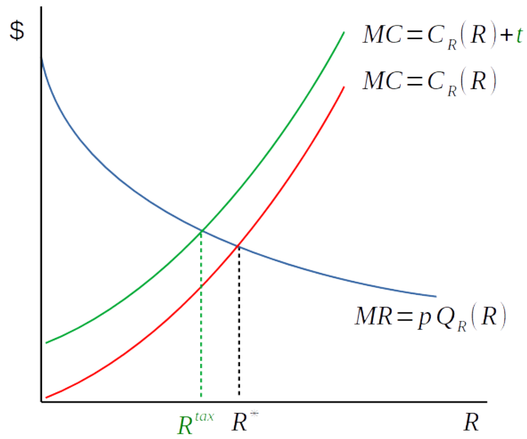


for R^* to be a global maximum.

Answer to (c) next page

(c) Let R^* be the profit maximizing level of resource extraction. Show that resource extraction declines if a resource extraction tax is added, i.e., $R^{tax} < R^*$.

Answer: A resource extraction tax, t, gives the profit function: $\pi(R) = pQ(R) - C(R) - tR$ with the following FOC: $\pi_R(R) = pQ_R(R) - C_R(R) - t = 0$. This implies that the marginal costs shift upwards. New intersection between marginal revenues and marginal costs occurs where $R^{tax} < R^*$



(d) Let $\bar{R}$ denote the total amount of the resource. Why is it useful to formulate the constrained profit maximization problem with a "less than equal" rather than "equality" resource extraction constraint?

Answer: Suppose we formulated this problem as an "equality" constraint. Constraints always overrule the maximization (or minimization) part in a constrained optimization problem. This implies that with an "equality" constraint, the mining of R could have to continue even if mining $\bar{R}$ gives less profits than mining R^*.

Formally, the maximization problem becomes $\pi(R) = pQ(R) - C(R)$ subject to (s.t.) $(\bar{R} - R)$ which gives the following Lagrangian:

$\mathscr{L}(R) = pQ(R) - C(R) + \lambda(\bar{R} - R)$ which gives the following FOCs:

(i) $\dfrac{\partial \mathscr{L}(R)}{\partial R} = pQ_R(R) - C_R(R) - \lambda = 0$

(ii) $\dfrac{\partial \mathscr{L}(R)}{\partial \lambda} = \bar{R} - R = 0$

where (ii) forces $R = \bar{R}$. If our unconstrained solution was $R^* < \bar{R}$, the constrained solution would give lower profits. Therefore, it is better to use a "less than equal" constraint. This moves us into Kuhn-Tucker territory where the FOCs become:

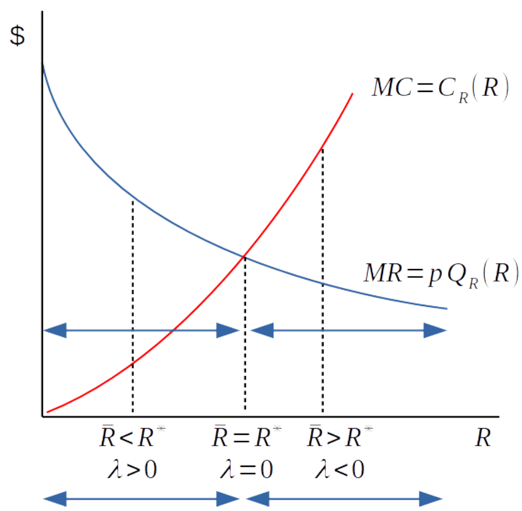
(i') $\dfrac{\partial \mathscr{L}(R)}{\partial R} = pQ_R(R) - C_R(R) - \lambda = 0$

(ii') $\dfrac{\partial \mathscr{L}(R)}{\partial \lambda} = \bar{R} - R \geq 0$

If (ii') is not binding, $\lambda = 0$ and (i') simplifies to $pQ_R(R) - C_R(R) - \lambda = pQ_R(R) - C_R(R) = 0$, which is the FOC of the unconstrained problem.

Remark 1: Let R^* be the solution of the unconstrained optimization problem, and let $\bar{R}$ be the required resource extraction (mining of R). Interpretation of λ (the constraint multiplier) in an "equality constrained" profit maximization problem:

1. $\lambda > 0 \Rightarrow R^* > \bar{R}$: The constraint reduces profits as the constraint "bites" (reduces the extraction of R)
2. $\lambda = 0 \Rightarrow R^* = \bar{R}$: The constraint (accidentally) coincides with the profit maximizing extraction level R'. Profits not affected by the constraint.



3. $\lambda < 0 \Rightarrow R^* < \bar{R}$: Forced extraction above the profit maximizing extraction level, *R'*. This also reduces profits. This take place if $MR(\bar{R}) - MC(\bar{R}) < 0$.

Remark 2: The equality constraint coincides with ordinary Lagrangian optimization. While I do not require you to know Kuhn-Tucker optimization for test 2, it is useful to know what it provides in terms of increased flexibility and what that implies:

1. The ≤ constraint is non-binding → the shadow price of the constraint (the equivalent of the Lagrangian multiplier) is zero.
2. The "≤ constraint" is binding → the constraint becomes an "equality constraint" which transfers the problem to a standard Lagrangian optimization problem, with the ordinary interpretation of the shadow price of the "equality constraint".

(EX4-2) A basic dynamic resource extraction model

Assume to technological progress, i.e., the production and cost functions do not change over time. This enables me to drop time subscripts in the production and cost functions to respectively write $Q(R_t)$ and $c(R_t)$. Note that production and costs may still vary over time,

My setup of the dynamic model based on my suggested solution of (1) then becomes:

(i) $\begin{Bmatrix} MAX \\ R_t \end{Bmatrix} NPV(R_t) = \begin{Bmatrix} MAX \\ R_t \end{Bmatrix} \int_0^\infty \pi_t(R_t) e^{-rt} dt = \begin{Bmatrix} MAX \\ R_t \end{Bmatrix} \int_0^\infty (p_t Q(R_t) - c(R_t)) e^{-rt} dt$

(a) Explain why is the profit maximizing quantity to harvest R^* can be given an expression that resembles the static optima solution when prices are constant ($p = p_t \forall t$), i.e., we get the FOC $\frac{\partial \pi_t}{\partial R_t} = pQ_R(R) - C_R(R) = 0$.

   Answer: The quick response to this is to set *r* in the discounting term to zero. That simplifies matters a lot, and you get a better "feel" for the question. In this particular case, this simplification is particularly useful:

   The blue term in (i) equals the static optimal solution except for the subscript *t*., while the red term captures the discounting effect. There is no linkages between the time periods in (i), the red discounting term is to be treated as a constant, This implies that the blue term is the only part of (i) that matters in the optimization.

(b) Formulate a constraint where the accumulated extraction of resources is less than or equal to the total availability of resources.

   Answer: In a dynamic setting resources, *R*, should last for the time span in consideration. This gives $\bar{R}$. From EX4-1 (d) we already know that resource use constraints generally should be of the "less than equal" form. The reason for this is that an "equality" formulation would force emptying the resource during the optimization time span even if this reduces profits and hence welfare. This gives the resource use constraint $\bar{R} = \int_{t=0}^{\infty} R_t dt$. Note that generally the time boundaries in the dynamic welfare maximization problem (i) must be the same as for the resource use constraint (here 0 and ∞).

(c) Intuitively, why would the addition of this constraint affect the solution in (a)?

   Answer: Because the resource constraint in (b) links the time periods: for example what is extracted earlier, reduces the potential extraction later. This implies than with this kind of resource constraint, we move from single time static optima to dynamic optima over the planning horizon of the optimization problem.