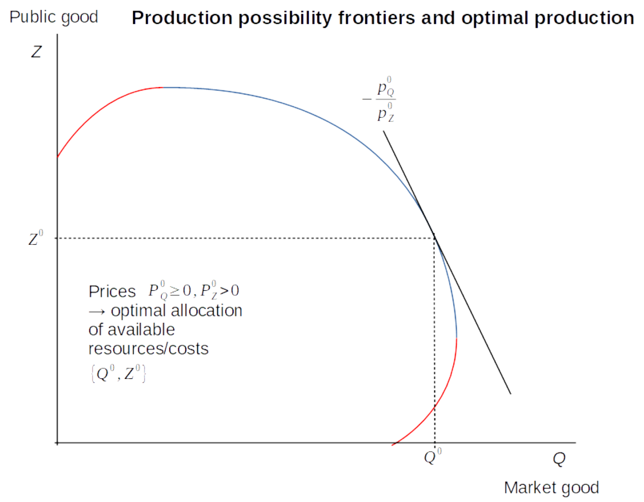
# ECN 275/375: Welfare enhancing production of public goods

**(EX1) Case 2: Public goods from agriculture**




Starting premise is the principal graph for the optimal provisioning of public goods from agriculture (right).

The intro.note explains the relarive price implications, but does not answer how we find those prices for public goods.

Pure public goods are non-exclusionary and non-rival in consumption, which means that a market cannot generate a meaningful price for the public good, *Z*. Recall the following table from your intro. course(s) in natural resource and environmental economics (ECN 170 at NMBU):

| | **Rival in consumption** | **Non-rival in consumption** |
|---|---|---|
| **Exclusionary in consumption (can exclude other users)** | Pure private good (ex. A pint of beer) | Club good (ex. Cable TV) |
| **Non-exclusionary in consumption (cannot exclude other users)** | Open access goods (fishing outside jurisdiction) | Pure public good (ex. A nice view) |

Market allocation through the price mechanism is only feasible for goods that are exclusionary in consumption. Note (not related to the question we are to answer) that market prices for the club goods may be due to market power, i.e., the resulting prices for club goods may not be those that maximize welfare.

This implies that provision of the (pure) public good needs to be paid for by taxes, *T*., and where *Q* is the private good, *Z* is the public good, *M* is money income, and $P_Q$ is the market price for the private good *Q*. Recall that the price of the public good, $P_Z$ in the budget constraint is zero which imples it is omitted. Start with the familiar model for utility maximization:

$$\underset{\{Q,Z\}}{MAX}\; U(Q,Z)\; s.t.\; M-T-P_Q Q=0$$

When we insert that production for the public good is paid for by taxes, i.e., $Z=z(T)$ (choose the tax *T* that gives the utility maximizing production of *Z*), we get:

$$\underset{\{Q,T\}}{MAX}\; U(Q,z(T))\; s.t.\; M-T-P_Q Q=0$$ (note that the choice variable *Z* has been replaced by *T*)

The Lagrangian becomes:

$\mathscr{L}=U(Q,z(T))+\lambda(M-T-P_Q Q)$ which we differentiate with *Q*, *T*, and $\lambda$ which gives us three equations:

(1) $\frac{\partial \mathscr{L}}{\partial Q}=\frac{\partial U(Q,z(T))}{\partial Q}-\lambda P_Q=0$

(2) $\frac{\partial \mathscr{L}}{\partial T}=\frac{\partial U(Q,z(T))}{\partial Z}\frac{\partial Z}{\partial T}-\lambda=0$ (chain rule → first differentiate by the Z-function, then by *T*)

(3) $\frac{\partial \mathscr{L}}{\partial \lambda}=M-T-P_Q Q=0$

Rearrange (1) and (2) to:

(1') $\frac{\partial U(Q,z(T))}{\partial Q} = \lambda P_Q \Rightarrow \frac{1}{P_Q} \frac{\partial U(Q,z(T))}{\partial Q} = \lambda$ (as $P_Q \neq 0$ this is OK ← do not divide by zero)

(2') $\frac{\partial U(Q,z(T))}{\partial Z} \frac{\partial Z(T)}{\partial T} = \lambda$ (remark: the marginal utility of spending funds should equal the shadow price, $\lambda$ , of the budget constraint, *M*).

Set (1') and (2') equal to each other to get

(4) $\frac{1}{P_Q} \frac{\partial U(Q,z(T))}{\partial Q} = \frac{\partial U(Q,z(T))}{\partial Z(T)} \frac{\partial Z(T)}{\partial T}$

the inverse price of the market good times the marginal utility of consumption of the private good equals the marginal utility of the public good times the production of the public good with increased effort (captured by taxes *T*). This implies that at the margin the marginal utility of private goods consumption should equal the marginal utility of spending taxes to produce the public good (a standard "no arbitrage" result characterizing optimal distribution of spending allocations).

To sum up, regardless of the budget, *M*, (4) characterizes the optimal trade-off between consumption of the private good *Q* and the public good *Z*.

With the assumption of decreasing marginal utility of money income (from lecture 3) we can deduce that rich societies (high *BNP/capita*) all other things equal, will produce more of the public good than poor societies (low *BNP/capita*).