ECN 275/375 Environmental and natural resource economics
Exercise set 17 – Eirik's suggested answers

Exercise 17.1 – The timber (stumpage) forestry model: Even aged stand management

In the basic forest model for even aged stand management, all the forest owner benefits of harvesting (clear cutting an area) comes at the end of the forest rotation, *T**.

(a) Given that the expected timber price at the time of harvest is $\hat{P}_{T^*}$, write down an equation for forest owner profits from timber harvesting when initial planting costs are k_0. Explain the terms in the equation, and briefly explain the reasoning behind your profit equation.

Answer: With all benefits (profits) occurring at the time of the (tree) harvest, the net present value of timer harvesting can be written as:

$\begin{Bmatrix} MAX \\ T^* \end{Bmatrix} \pi_{\hat{P}_T} = \begin{Bmatrix} MAX \\ T^* \end{Bmatrix} \hat{P}_{T^*} S(T^*) e^{-iT^*} - k_0$, where *i* is the forest owner's individual discount rate.

(b) Solve for the optimal rotation age *T**, and interpret your solution.

Answer: Differentiate your expression in (a) with respect to *T**. Given my specification:

$$\frac{\partial \pi_{T^*}}{\partial T^*} = \hat{P}_{T^*} \dot{S}(T^*) e^{-iT^*} + \hat{P}_{T^*} S(T^*)(-i\, e^{-iT^*}) = 0 \Rightarrow \frac{\dot{S}(T^*)}{S(T^*)} = i \quad \text{(all green terms cancel out)}$$

This implies that at the optimal rotation age *T**, the *volume growth of the tree biomass over time* equals the forest owner's individual discount rate.

Remark: From economics in general we know that benefits should cross costs from above (think in terms of a supply and demand framework). The interest rate line represented by *i* is the cost side (opportunity costs of harvesting = supply), while the *volume growth of the tree biomass over time* represents the benefit side (demand).

(c) Suppose that planting at the costs k_0 in (a) shortens the optimal rotation age by 20 years. How would you determine if planting increases or decreases the net benefits to three owners.

Answer: From (a) we already have the equation for expected profits under planting. The equation without planting is and with 20 years longer rotation period:

$$\pi_{\hat{P}_{T^*+20}} = \hat{P}_{T^*+20} S(T^*) e^{-i(T^*+20)}$$

The optimal strategy (planting at costs k_0 or no planting withouth planting costs) now depends upon which of the two values is the largest. Planting is profitable if:

$$k_0 < (\hat{P}_{T^*} S(T^*) e^{-iT^*}) - (\hat{P}_{T^*+20} S(T^*) e^{-i(T^*+20)})$$

Remark: The optimal rotation age is just extended by 20 years as planting does not influence the optimal rotation age in (a) as there is no k_0 in the characteristics of the optimal rotation age solution $\frac{\dot{S}(T^*)}{S(T^*)} = i$.

(d) Moose primarily graze on on younger aged tree stands or recent clear cuts. How is the inclusion of moose hunting revenues going to affect the decision to plant or not in (c)? Respond verbally, but provide a short reasoning for your answer.

Answer: No planting extends the period of the rotation where the acreage is not covered by extensive tree growth (= better growing conditions for tree species the moose likes → improved grazing conditions for the moose). Hence, the period of suitable moose habitat is extended, and planting becomes relatively less profitable than in the case where only the profits from timber harvests are included.

(e) If the benefits of carbon sequestration are added, how would that influence the planting decision relative to (d)? Respond verbally, but provide a short reasoning for your answer.

Answer: It is reasonable to assume that the amount of carbon sequestered is linked to the per volume growth of tree biomass, $\frac{\dot{S}(t)}{S(t)}$. As planting moves the high volume growth of tree biomass forward in the rotation period, planting will increase the mean carbon sequestration in the rotation period compared to no planting.

Remark: Given that we struggle to reduce carbon emissions in the short to medium run, increasing carbon sequestration early in the rotation period by planting, reduces net carbon emissions in the short to medium run, i.e., provides earlier reductions in net emissions.

Exercise 17.2 – Numerical example of the single stand forest management problem

A stand of eucalyptus grows according to the following Cobb-Douglas function in *t* per hectare:

$$S(t)=A(bt)^{\alpha}=10\left(\frac{1}{3}t\right)^{\frac{2}{3}}$$

where $S(t)$ is the tree volume in m^3 at time *t* (in years), and *A*=10 is a scale parameter. The expected cost of replanting a hectare of forest is $\hat{k}_{T^*}=2000$, the expected net price of timber (harvest value less harvest costs) per m^3 at the optimal time of is $\hat{P}_{T^*}=200$, and the forest owner's individual discount rate *i* = 3%).

(a) Write down an expression for the expected per hectare profits (rents) at time *T**.

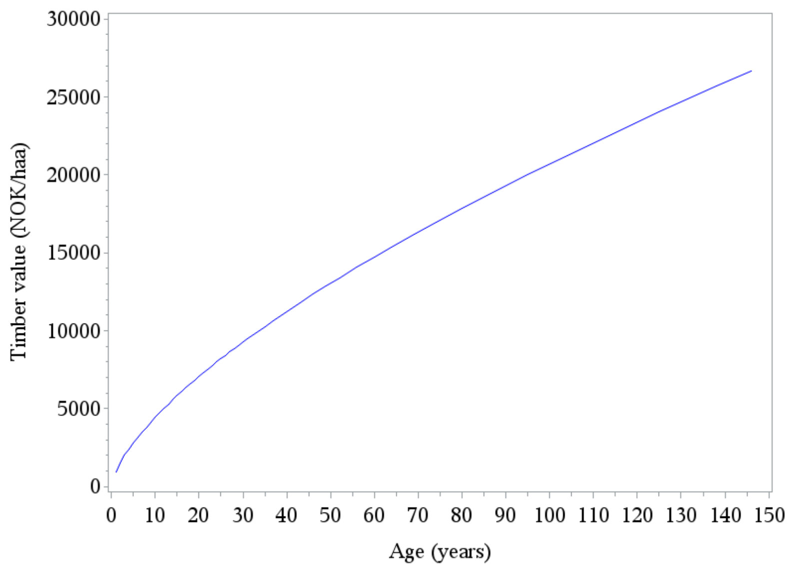
Answer: Note that this is the *replanting costs* at the end of the (optimal) rotation age. Compared to the standard problem in 17.2, the answer is slightly different (we also discount the replanting costs):

$$\pi_{\hat{P}_T}=\left(\hat{P}_{T^*}S(T^*)-k_{T^*}\right)e^{-iT^*}=\left(\hat{P}_{T^*}A(bT^*)^{\alpha}-k_{T^*}\right)e^{-iT^*}=\left((200)10\left(\frac{T^*}{3}\right)^{\frac{2}{3}}-2000\right)e^{-0.03T^*}$$

(b) Graph the expected nominal per hectare revenues (gross income) as an expression of tree age, *t*.

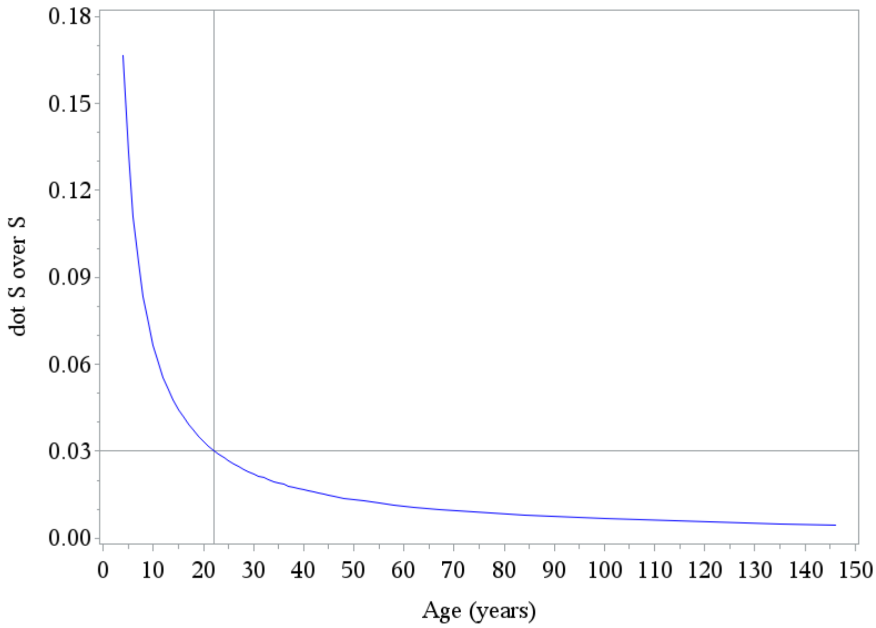
Answer: This is the graph of $(200)10\left(\frac{t}{3}\right)^{\frac{2}{3}}$

Remark: Note that this graph is almost linear.



(c) With the individual discount rate $i = 3\%$, show graphically that the optimal rotation age is about 22 years.

Answer: Graph $\frac{\dot{S}(t)}{S(t)}$, where using the core rule for differentiation gives $\dot{S}(t)=\alpha b A(bt)^{\alpha-1}=\frac{2}{3}\frac{1}{3}(10)(\frac{1}{3}t)^{-\frac{1}{3}}$. This gives the following graph:



where we see that the growth per volume curve $\frac{\dot{S}(t)}{S(t)}$ crosses the interest rate line from above at approximately 22 years.

(d) Solve for the optimal rotation age.

Answer: From exercise 17.1.a we know that the optimal rotation age is given by $\frac{\dot{S}(T^*)}{S(T^*)}=i$.

This gives $\frac{\dot{S}(t)}{S(t)}=\frac{\frac{2}{3}\frac{1}{3}(10)(\frac{1}{3}t)^{-\frac{1}{3}}}{10(\frac{1}{3}t)^{\frac{2}{3}}}=\frac{\frac{2}{9}}{(\frac{1}{3}t)^{\frac{2}{3}+\frac{1}{3}}}=\frac{2}{3}\frac{1}{t}=0.03\Rightarrow t=\frac{2}{3}(\frac{1}{0.03})=22.2$

i.e., 22.2 years.

Remark: At the first look this exercise seems to involve a lot of calculations, but many items cancel out quite nicely. My advice is to work with the letter symbols as long as possible to avoid numeric mistakes. Also, for processing data on a computer, working with letter symbols that one attaches values to later, makes it easier to enter correctly, makes the program (or spreadsheet) more transparent, and increases flexibility for later use (easier reparameterization).