ECN 275/375 Environmental and natural resource economics
Exercise set 16 – Eirik's suggested answers

Exercise 16.1 – Optimal extinction of a fish species

A fishery is not in steady state, i.e., the optimal effort, E_t^* has not been identified, and hence the optimal harvest may vary over time. Moreover, fish prices, P_t, and effort costs, w_t, are allowed to vary over time when r is the discount rate that is assumed constant over time.

(a) Write down the discrete and continuous versions of the NPV formula for this fishery for infinite time.

Answer:

Discrete time: $\begin{Bmatrix} MAX \\ E_t \end{Bmatrix} NPV = \begin{Bmatrix} MAX \\ E_t \end{Bmatrix} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left(P_t H(E_t) - w_t E_t\right)$

Contin. Time: $\begin{Bmatrix} MAX \\ E(t) \end{Bmatrix} NPV = \begin{Bmatrix} MAX \\ E(t) \end{Bmatrix} \int_{t=0}^{\infty} \left(P(t) H(E(t)) - w(t) E(t)\right) e^{-rt} dt$

(b) Assume it becomes optimal to harvest a fish species to extinction at some finite time T'. State the verbal condition for fish extinction to be optimal, and write down the mathematical conditions.

Answer: The net present value of harvesting more than the sustainable yield of the fish for the time T' exceeds the infinite value of fish harvests that are sustainable, i.e., the fishery can go on forever.

Denote the NPV formulas in (a) NPV_{sust} to simplify the equations.

Discrete time: $\begin{Bmatrix} MAX \\ E_t \end{Bmatrix} NPV = \begin{Bmatrix} MAX \\ E_t \end{Bmatrix} \sum_{t=0}^{T'} \left(\frac{1}{1+r}\right)^t \left(P_t H(E_t) - w_t E_t\right) > NPV_{sust}$

Contin. time: $\begin{Bmatrix} MAX \\ E(t) \end{Bmatrix} NPV = \begin{Bmatrix} MAX \\ E(t) \end{Bmatrix} \int_{t=0}^{T'} \left(P(t) H(E(t)) - w(t) E(t)\right) e^{-rt} dt > NPV_{sust}$

(c) The outcome in (b) becomes more likely the higher the discount rate. Explain why this is the case.

Answer: The higher the discount rate, the less weight are given to the profits for time > T', which may make it more profitable to increase fish harvests early at the expense of making the fish species extinct by early over harvesting)

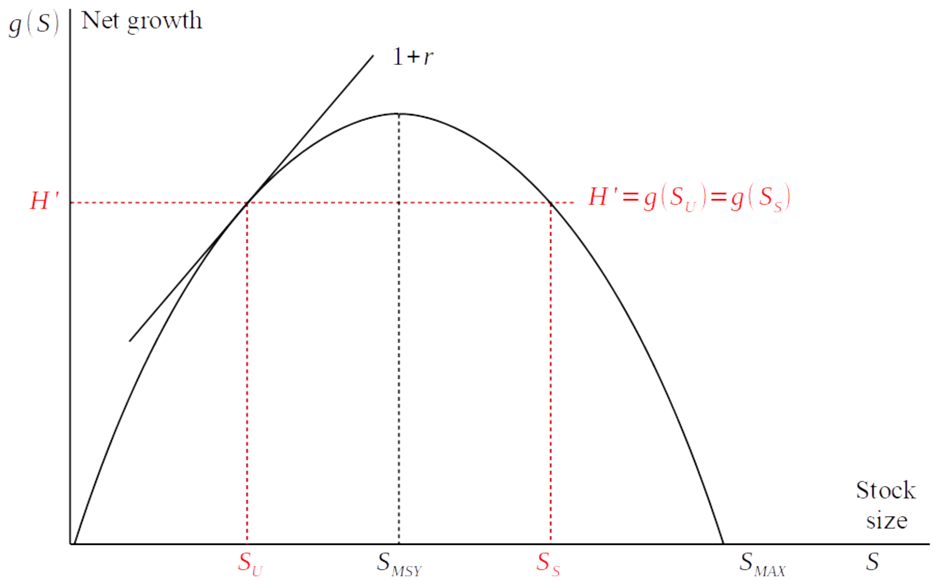
Remark: This is most easily seen in the discrete time version of the formula, where the term $\frac{1}{1+r}$ declines more rapidly for higher values of r ($\frac{1}{1+r} > \frac{1}{1+r_H}$ when $r_H > r$). This effect becomes particularly strong as the number of time periods increases in the NPV-formula.

(d) Suppose a monopolist controls the fish harvest of a species. How may that reduce the risk of over harvesting and hence extinction.

Answer: A monopolist is more likely to try to harvest about the same amount each year, which reduces the risk of early over harvesting and hence extinction. To see this, recall that for the monopolist the market price for fish is a function of the (yearly) harvest level, i.e., $P_t(H(E_t))$.

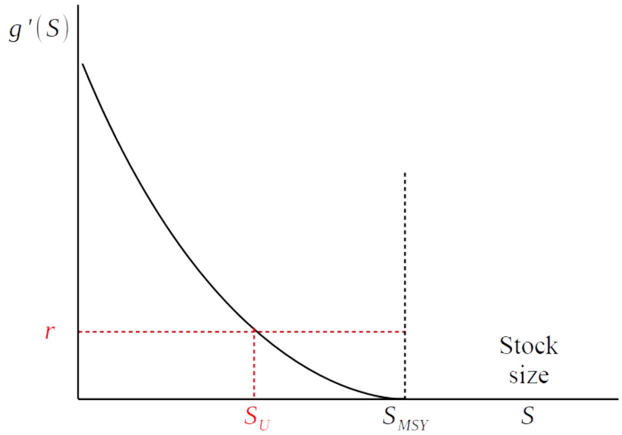
Exercise 16.2 – Optimal extinction and safe minimum standards

Consider a "standard" bell growth function for a fish species. Assume there is no uncertainty regarding the growth function or the stock size. Let r denote the risk free return on capital, which in discrete time gives the capitalization factor $(1+r)$. The figure below illustrates a situation where the unstable stock-harvest equilibrium, $\{S_U, H'\}$ is marked, and where the capitalization factor line tangents the growth function $G(S)$.



(a) Draw $G'(S)$ and r in the same graph based on the above figure for $S < S_{MSY}$. Based on the graph suggest why the "fish as capital" perspective gives S_U as the optimal stock level.

Answer:



The growth function is concave. This implies that for $S < S_{MSY}$, its derivative $G'(S)$ is declining but positive.

As r and $G'(S_U)$, the returns from letting the money grow in the bank and at the harvest H' are equal. For $S > S_U$ $G'(S) < r$, while for $S < S_U$ $G'(S) > r$. This suggests S_U is the optimal stock level.

(b) Explain the result in (a) verbally and mathematically.

Answer: The net value of harvesting all of the fish and putting the net revenues in the bank (the risk free alternative) is pS which for the following years would grow at the interest rate r forever. Let p equal the net price. The yearly capital gains are therefore rpS, which gives yearly marginal revenues of the stock rp.

Now consider harvesting H', a steady state harvest level, which gives the yearly net revenues $pH' = pG(S)$ forever. This gives yearly marginal revenues $pG'(S)$.

Setting the two marginal revenue streams equal to each other gives $rp = pG'(S)$ which after canceling out p on both sides gives $r = G'(S)$. Given the way the figure is drawn, we get the optimal stock level S_U as $r = G'(S_U)$.

Remark: This result hopefully makes it easier to interpret the Farmer-Randall result (see figure in lecture note 15)

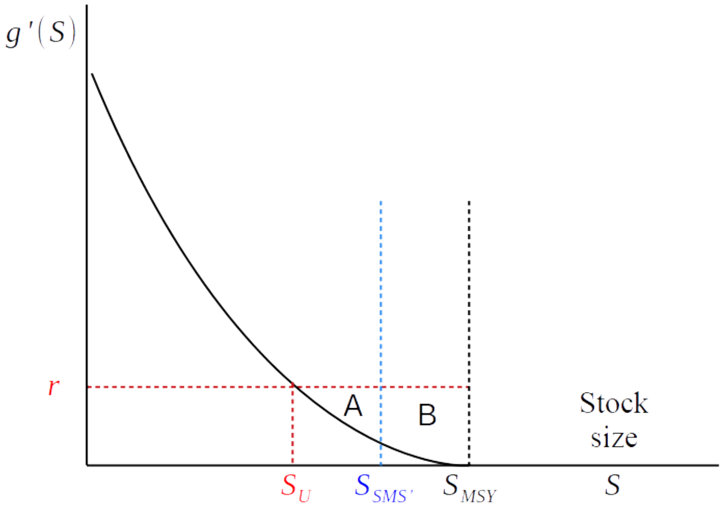
(c) Now suppose that a safe minimum standard is set such that $S_{SMS} > S_U$. (i) Show graphically that the economic loss of the safe minimum standard grows as $S_{SMS} - S_U$ grows. (ii) What does that tell about capital losses safe minimum standards.

Answer: (i) Consider two safe minimum standards:

$S_{SMS'}$ which gives the capital loss A

SMSY which gives the capital loss A+B

(ii) Setting a two tough SMS, here requiring the stock level to deviate from the capital asset optimum, S_U, increases capital losses.



Remark: As all harvest-stock equlibria $\{H, S\}$ where $S \leq SMSY$ are unstable, the financial gains of a stock level close to S_U entails risks the fish stock will be driven to zero. That implies that the revenue streams from fishing will vanish, which we already have seen is sub-optimal in (a). The more precise knowledge we have about the stock level and the growth function, the closer the SMS can be to S_U.