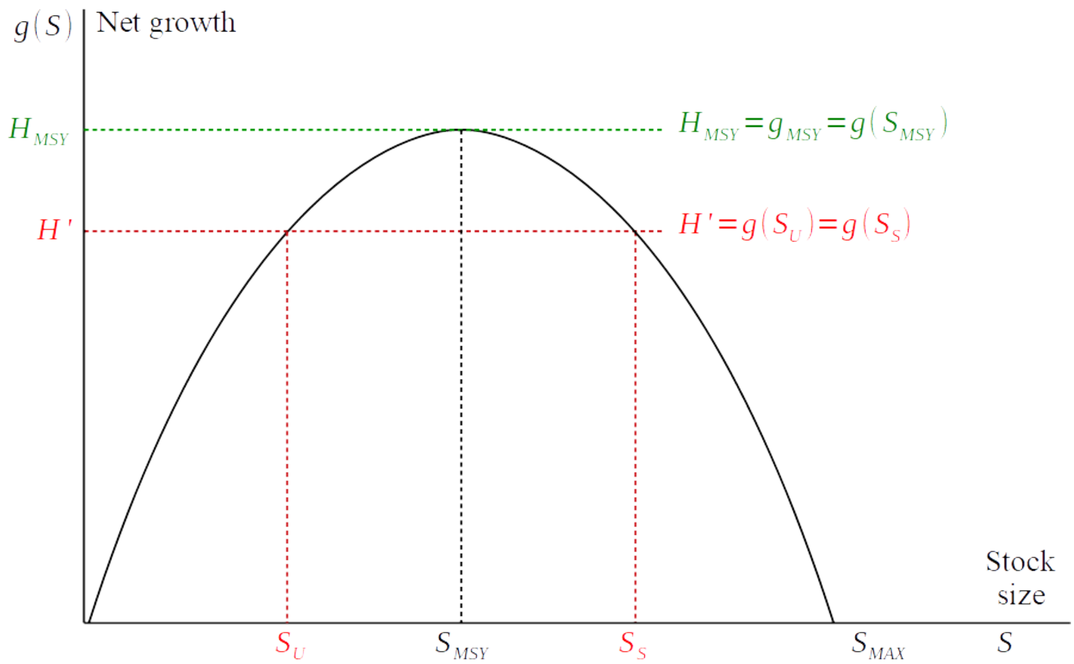
ECN 275/375 Environmental and natural resource economics
Exercise set 15 – Eirik's suggested answers

Exercise 15.1 – Properties of the stock-net growth relationship

In fisheries economics and management, the relationship between stock size, *S*, of a fish species and the net growth (birth minus natural deaths), $g(S)$ in biomass is important to understand the impacts of fishing (harvesting) on the fish species.

(a) Draw a typical *stock size - growth in stocks* graph, and label the axes. Identify the maximum sustainable yield stock, S_{MSY}, and net growth $g_{MSY}=g(S_{MSY})$, and the maximum stock size, S_{MAX}.

Answer: The typical graph of such a fish species below (black and green, red is for (b))



(b) Introduce a sustainable harvest level, *H'*. In the same graph as in (a), mark the stock size-harvest equilibria, and discuss their stability. What could theoretically happen to the fish stock if the the harvest level is maintained at *H'* and the stock size is less than S_U.

Answer: Additional graphical elements in red, where there are two equilibrium stock sizes, S_U and S_S.

The equilibrium $\{S_S, H'\}$ is **stable** because for $S_S > S > S_U$, $g(S) > H' \rightarrow$ the stock size increases to S_S, and for $S < S_S$, $g(S) < H' \rightarrow$ the stock size decreases to S_S.

The equilibrium $\{S_U, H'\}$ is **unstable** because for $S_S > S > S_U$, $g(S) > H' \rightarrow$ the stock size increases to S_S, and for $S < S_U$, $g(S) < H' \rightarrow$ the stock size decreases to zero.

If the harvest level is maintained H' and the stock size is less than S_U it follows from the stability analysis that the stock size could in theory go to zero. In practice, that would most likely not happen as it would be difficult to maintain the harvest level H' for low stock sizes. In practice, the stock size would decline and the fish species is utmost vulnerable to extinction.

(c) What would entail a virgin fishery, i.e., a fishery on a fish stock that never had been subject to fishing before?

Answer: The introduction of fishing ($H > 0$), implies that the stock size is reduced to the stock size that is in equilibrium with the harvest level. For example if we on an unfished species introduced ther harvest lever H' in the figure, fish stocks would decline to S_S, where we already have shown that S_S, H' us a stable equilibrium.

(d) Indicate the maximum sustainable harvest level, H_{MSY}, and discuss the stability properties of the ensuing equilibrium. Given your stability analysis, discuss the desirability using maximum sustainable harvest as an objective for managing a fishery.

Answer: H_{MSY} is drawn by the green line. The maximum sustainable yield stock size, S_{MSY}, is the only stock size that gives a feasible equilibrium { S_{MSY}, H_{MSY} }. For stock sizes different from S_{MSY}, the harvest $H_{MSY} > g(S_{MSY})$, which implies that the equilbrium { S_{MSY}, H_{MSY} } is unstable.

This implies that erroneous estimations of the maximum sustainable yield stock, $\hat{S}_{MSY} \neq S_{MSY}$, or too high estimates of the maximum sustainable net growth, $\hat{g}(S_{MSY}) > g(S_{MSY})$, may lead to declining stock sizes, and hence a decline in the maximum sustainable yield. Hence, the harvesting strategy H_{MSY} may not be sustainable (sustainability defined as non-declining future harvest yield levels).

Remark: Maximum sustainable yield harvest, H_{MSY}, may not coincide with the harvest level that maximizes rents from the fishery, and hence not economically optimal (see exercise 15.2).

Exercise 15.2 – Mathematical and graphical analysis of a steady state fishery

Fisheries are usually framed in an effort-harvest dimension, where harvests, H_t are framed as a function of effort, E_t. Assume a steady state equilibrum, i.e., fish stock sizes S_t and the net growth as a function of stocks, $g_t = g(S_t)$ are unchanged over time.

(a) Write a mathematical model for maximizing the rents (profits) from a steady state fishery. Be clear on what is the decision variables.

Answer: In steady state the stock size and the net growth of the fish stock (population) are constant. Let P denote the price on the fish, and w the costs per unit of effort, E., where effort is the decision variable. The static equilibrium profit maximizing model then becomes:

$$\begin{Bmatrix} MAX \\ E \end{Bmatrix} \pi = \begin{Bmatrix} MAX \\ E \end{Bmatrix} PH(E) - wE$$

(b) State the necessary first and second order conditions finding the profit (rent) maximizing solution. Interpret the conditions.

Answer: Differentiate the expression in (a) with respect to the choice variable, effort to get:

First order condition: $\frac{\partial \pi}{\partial E} = \frac{\partial}{\partial E}(PH(E) - wE) = PH_E - w = 0$ som kan skrives som $H_E = \frac{w}{P}$.

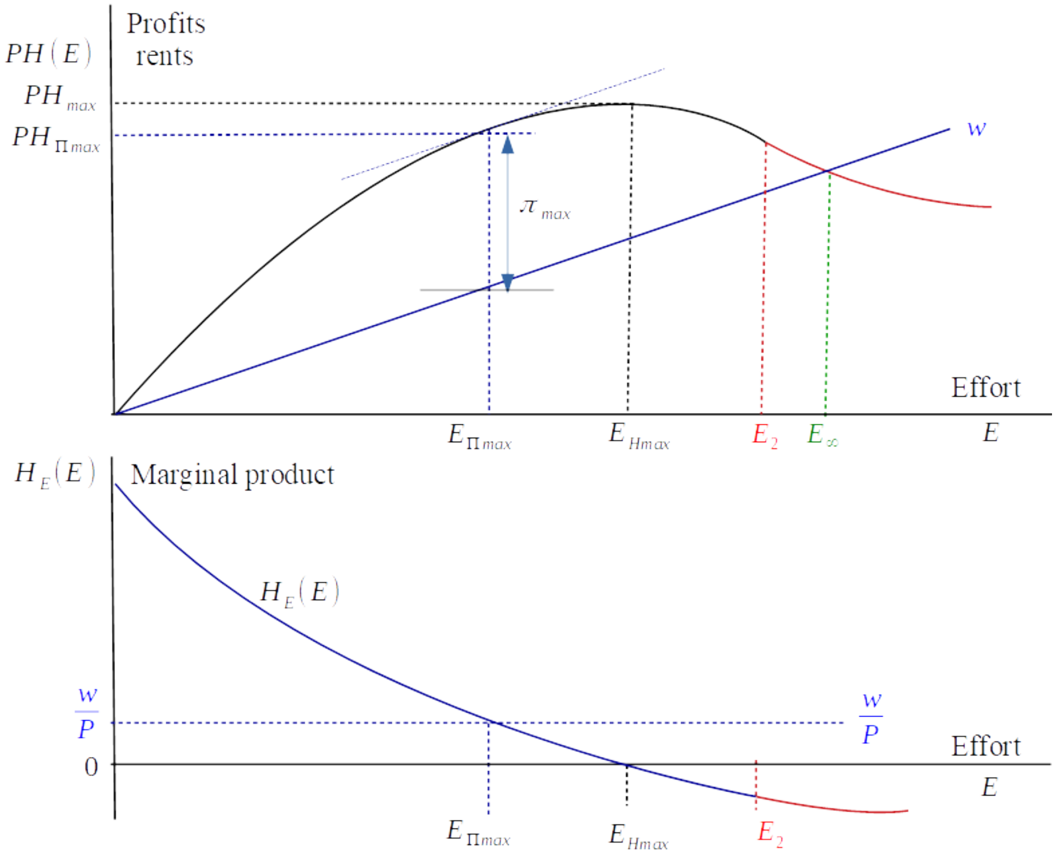
This is the marginal revenue PH_E less marginal costs w is zero. With a positive per unit cost of effort, $w > 0$, H_E must be positive around E^* for the condition to hold (with positive price P).

Second order condition: $\frac{\partial^2 \pi}{\partial E^2} = \frac{\partial}{\partial E^2}(PH_E - w) = PH_{EE} < 0$

With a positive price P the double derivative of the harvest function $H(E)$, $H_{EE} < 0$, i.e., declining marginal productivity of effort.

(c) Draw a graph of the equation in (a) that is consistent with the conditions in (b). Mark regions where the necessary conditions hold (hint: they need only to hold locally around $E_{\Pi max}$).

Answer: The marginal product H_E only needs to be positive (first order condition) around the optimum point $E_{\Pi max}$ in the figure), while the second order condition ($H_{EE} < 0$) needs to hold to avoid local optima. The graphs under show the total and marginal products with the conditions met locally (around $E_{\Pi max}$). Note that for the red segment: $H_{EE} > 0 \rightarrow$ effort in this region no candidate for profit maximizing effort levels)



Remarks:

1. It is analytically easier to use the formulation of first order condition with relative prices, i.e., $H_E = \frac{w}{P}$. One reason for this is that an increased price P would lower the blue price line and increase the profit maximizing effort (and higher costs for effort would shift the relative price line up $\rightarrow$ the optimal effort decreases).
2. The marginal product becomes negative for effort levels beyond $E_{\Pi max}$.

(d) How would you find the open access effort, E_∞. Explain your reasoning.

Answer: The open access equilibrium E_∞ is defined where the profits (rents) are zero, i.e., solve for E_∞:

$e_\infty : PH(E) - wE = 0$ (or in a less formal formulation: $PH(E_\infty) - wE_\infty = 0$).

The reason for this is that there is free entry (everybody who wants, can enter the fishery – kind of flollows from the term *open access*). Hence, effort would increase until profits (rents) become zero. Remarks:

1. When harvesting a natural resource there are extra rents, which are the value of "nature's contribution", which is unpaid)
2. Fishermen are still paid their wages, w (to see this check the profit function).