ECN 275/375 Environmental and natural resource economics
Exercise set 12 – Eirik's suggested answers

Exercise 12.1 – The "cake eating" problem

Henry is 70 years old and expects to live for ten more years. He is paranoid about the authorities and paying taxes, and has therefore stashed away 1 million NOK in a safe place. He is well aware that he forfeits interest income on this, but his paranoia is stronger.

Henry has a personal discount rate (impatience of consumption) of 10 percent per year. For simplicity assume Henry goes to the secret hiding place and fetches money once a year. His utility of consumption is the natural log function of the form *ln*(y+1) where 1 equals 10 000 NOK, and *y* is income in 10 000 NOK.

(a) Formulate Henry's decision problem.

Answer: $\begin{Bmatrix} max \\ y_t \end{Bmatrix} \sum_{t=0}^{9} U_t(y_t)\left(\frac{1}{1+\delta}\right)^t = \begin{Bmatrix} max \\ y_t \end{Bmatrix} \sum_{t=0}^{9} \ln(1+y_t)\left(\frac{1}{1+0{,}10}\right)^t$ subject to

$K_t = K_0 - \sum_{t=0}^{9} y_t = 100 - \sum_{t=0}^{9} y_t$ and $K_t \geq 0$ and $y_t \leq K_t$

(b) Build a simple simulation model on your computer, and try to find Henry's optimal consumption path for the ten years he expects to live.

Answer: See the table below. In the optimal solution we know that Henry should not have any capital left. We also know that the consumption levels should be declining over time. I tested with various levels of declining consumption (the adj column), and found that an adjustment factor of 0.951 worked quite well after testing with some alternatives. With 10 consumption periods the (close to) optimal solution should have consumption above 10 in the first year.

adj	Welfare	Time	Capital stock	y_in	y1	y2	y3	y4	y5	y6	y7	y8	y9	y10
0.951	14.2624	7	-2.7770	16.0	15.2160	14.4704	13.7614	13.0871	12.4458	11.8359	11.2560	0.0000	.	.
0.951	15.3519	8	-9.4270	15.5	14.7405	14.0182	13.3313	12.6781	12.0569	11.4661	10.9042	10.3699	0.00000	.
0.951	15.1739	8	-5.8971	15.0	14.2650	13.5660	12.9013	12.2691	11.6679	11.0962	10.5525	10.0354	0.00000	.
0.951	14.9902	8	-2.3672	14.5	13.7895	13.1138	12.4712	11.8601	11.2790	10.7263	10.2007	9.7009	0.00000	.
0.951	15.8705	9	-7.3083	14.0	13.3140	12.6616	12.0412	11.4512	10.8901	10.3565	9.8490	9.3664	8.90744	0.0000
0.951	15.6594	9	-3.4759	13.5	12.8385	12.2094	11.6112	11.0422	10.5011	9.9866	9.4972	9.0319	8.58931	0.0000
0.951	16.3832	10	0.3566	13.0	12.3630	11.7572	11.1811	10.6332	10.1122	9.6167	9.1455	8.6974	8.27119	8.2225
0.951	16.2944	10	4.1890	12.5	11.8875	11.3050	10.7511	10.2243	9.7233	9.2468	8.7937	8.3628	7.95307	11.7524
0.951	16.1634	10	8.0215	12.0	11.4120	10.8528	10.3210	9.8153	9.3343	8.8770	8.4420	8.0283	7.63494	15.2823
0.951	16.0031	10	11.8539	11.5	10.9365	10.4006	9.8910	9.4063	8.9454	8.5071	8.0902	7.6938	7.31682	18.8122
0.951	15.8192	10	15.6863	11.0	10.4610	9.9484	9.4609	8.9974	8.5565	8.1372	7.7385	7.3593	6.99870	22.3421

Remark: The optimal solution is close to the above solution, but with some capital stock left in the 10th time period (0.3556), he could have done a bit better if he had started the consumption in the first year a bit higher than at 13 (NOK 130 000) (the column *y_in*).

(c) Suppose that Henry does not know exactly when he will die. He is well educated in statistics, and figures that with his health record and eating habits, there is 95 % probability that that he could live 12 more years in stead of his initial estimate of 10 years. How would that change his consumption profile based on the above table..

Answer: Lower initial consumption, but still high enough that he spends all of his capital by the time he turns 82. That suggests an initial consumption of 12 (120 000 NOK). We also observe that his overall welfare drops quite sharply if he cannot make his savings last for as long as he expects to live.

Exercise 12.2 – Backstop technology

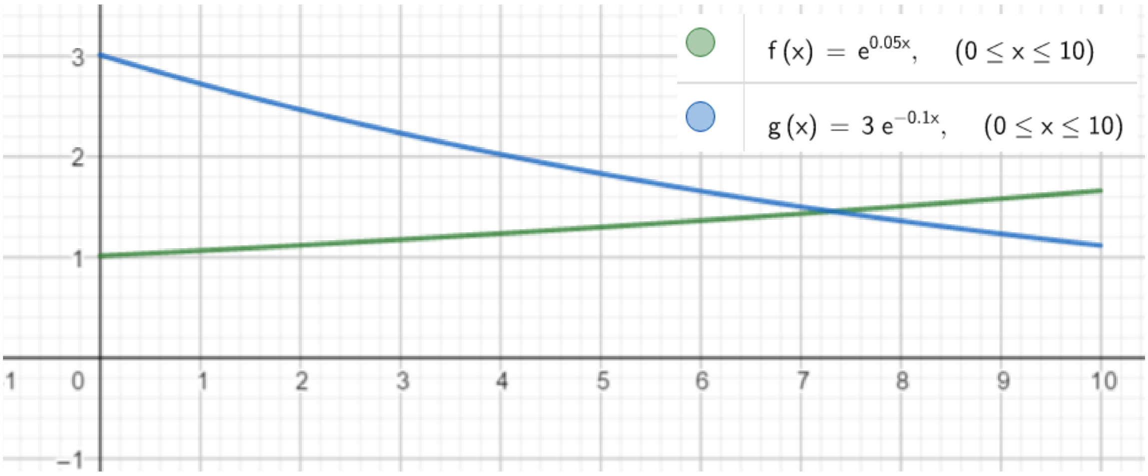
The initial per unit price of a resource is 1, and society's discount rate is 5%. In year zero, the price of the backstop technology is 3. Annually, the unit cost of using the new technology drops by 10%.

(a) At what time should the backstop technology introduced?

Answer: In optimum we know that the resource and backstop technology prices should be equal. This allows setting $1e^{rt} = 3e^{st}$ and solve for *t* by taking the log on both sides:

$$\ln(e^{rt}) = \ln(3e^{st}) \Rightarrow$$
$$rt\ln(e) = st\ln(3e) \Rightarrow$$
$$(r-s)t = \ln(3) \Rightarrow$$
$$t = \frac{\ln(3)}{r-s} = \frac{\ln(3)}{0{,}15} = 7{,}32$$

We can also check our solution using graphing software like GeoGebra.



(b) What would be the impacts of expectations about a backstop technology be on resource extraction and resource prices?

Answer: As the resource would be of little value once the backstop technology becomes available, resource extraction would increase. With larger quantities of the resource in the market place, the resource price would fall. In turn, that would delay the introduction of the new technology.