ECN 275/375 Environmental and natural resource economics
Exercise set 11

Exercise 11.1 – Isoquants

Two production functions are of the form $Q_i = Q_i(K, N)$, where K is man-made capital, N is natural capital, and $i \in \{1, 2\}$. Both production functions are regular with (standard) convex isoquants to the origon with , where the respective elsticities of substitution, σ_1 and σ_2, have the following values: $0 < \sigma_1 < \sigma_2 < \infty$. The input factor prices are δ and v for the two input factors.

(a) Draw the two isoquants for the same production level, $Q_1^0 = Q_2^0$, and show the resulting optimal uses of the two inputs.

(b) Suppose a tax t is added to the price of natural capital. From a producer's perspective, what is the price on the use of natural resources? Show graphically that the relative price change gives a larger input factor substitution from the price change for the production function that has the highest value for the elasticity of substitution (isoquant 2).

(c) Explain technically and intuitively why the cost increase of the tax is larger for the least substitution elastic production function (isoquant 1).

Exercise 11.2 – Cobb-Douglas example of unconstrained and constrained production

The production function is: $Q(K, N) = A K^{\alpha} N^{\beta}$ with the input factor K (man-made capital) and N (natural capital), and with $A > 0, 1 > \alpha > 0, 1 > \beta > 0$. The product price is $p > 0$, and the input factor prices are $\delta > 0, v > 0$ respectively for K and N.

(a) What is the profit function?

(b) Show that the input factor demand functions expressed in terms of the parameters and the prices (p, δ and v) becomes: $N(p, \delta, v)^* = \left(\frac{1}{pA}\right)^{\frac{1}{\alpha+\beta-1}} \left(\frac{\beta}{v}\right)^{\frac{\alpha-1}{\alpha+\beta-1}} \left(\frac{\delta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta-1}}$ and

$K(p, \delta, v)^* = \left(\frac{1}{pA}\right)^{\frac{1}{\alpha+\beta-1}} \left(\frac{\alpha}{\delta}\right)^{\frac{\beta-1}{\alpha+\beta-1}} \left(\frac{v}{\beta}\right)^{\frac{\beta}{\alpha+\beta-1}}$. **(not exam relevant)**

(c) Suppose that production is quantity restricted to $\bar{Q}$. Set up the appropriate optimization problem, the corresponding Lagrangian, and the first order conditions.

(d) Show that the conditional (quantity constrained) demand functions for the constraint $\bar{Q}$ are $K(\bar{Q}, \delta, v)^C = \left(\frac{\bar{Q}}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{v}{\delta}\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}$ and $N(\bar{Q}, \delta, v)^C = \left(\frac{\bar{Q}}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{\delta}{v}\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}$ **(not exam relevant)**

(e) The parameters in the production function: $A = 50$, $\alpha = 0{,}4$, and $\beta = 0{,}3$.

Prices: the product price $p = 5$, and the input factor prices are $\delta = 4$ (in percent) and $v = 6$ respectively for K and N. Find the profit maximizing input factor uses, K^* and N^*, the profit maximizing production quantity, Q^*, and the corresponding profits (hint: enter the equations with parameters instead of parameter values, and initialize the parameter values in a separate segment of your spreadsheet or computer program).

(f) Using the same prices as in (e) the table below shows the input uses, profits, and the adjusted shadow price (the Lagrangian multiplier) for the following quantity restrictions $\bar{Q}$ = 40 000 to 60 000 by intervals of 2 000.

Scenario	Profits	Quantity produced	Manmade capital	Nature capital	Shadow price
Profit maximization	68532	45688	22844	11422	.
Restriction Q = 30000	62320	30000	12526	6263	-0.396
Restriction Q = 32000	63852	32000	13735	6868	-0.342
Restriction Q = 34000	65154	34000	14978	7489	-0.289
Restriction Q = 36000	66233	36000	16252	8126	-0.237
Restriction Q = 38000	67098	38000	17557	8779	-0.186
Restriction Q = 40000	67754	40000	18892	9446	-0.137
Restriction Q = 42000	68208	42000	20256	10128	-0.088
Restriction Q = 44000	68464	44000	21648	10824	-0.040
Restriction Q = 46000	68529	46000	23067	11534	0.007
Restriction Q = 48000	68407	48000	24513	12257	0.054
Restriction Q = 50000	68103	50000	25985	12993	0.099
Restriction Q = 52000	67621	52000	27483	13741	0.144
Restriction Q = 54000	66964	54000	29005	14503	0.189
Restriction Q = 56000	66137	56000	30552	15276	0.232
Restriction Q = 58000	65142	58000	32123	16061	0.275
Restriction Q = 60000	63984	60000	33717	16858	0.318

Comment on the following

i. Based upon the adjusted shadow price values, what appears to approximately be the optimal production level, Q?

ii. Any other information in the table that can help you to approximately identify the optimal production level Q except the first row? If so, what and why?

iii. The optimal input use combinations are in a fixed ratio to each other. Explain why that is the case.

iv. The profits decline for production levels above 46 000. What is the reason for that?