ECN 275/375: Natural resoruce and environmental economics
Exercise set 10 - Eirik's suggested answers

Exercise 10.1: Non-cooperative games and cooperative outcomes

Consider a game that is repeated with random stop time with five players. Their payoffs from various strategies are displayed in the table below. All payoffs are common information (= all players know the payoffs of all players). The common discount rate is 5%.

Strategy for player *i* | strategy at least one other player	Player				
	A	B	C	D	E
do not cooperate | cooperate	135	140	145	150	155
cooperate | cooperate	125	125	125	125	125
do not cooperate | do not cooperate	100	100	100	100	100
cooperate | not cooperate	50	60	70	80	90

(a) Show that this game does not have a cooperative solution given these payoffs.

Answer: Given the discount rate, we know that the cut off point for all players for inducing a cooperative solution is given at 1/(1+0.05) = 0.952. So, we test if the Folk theorem formula holds for each of the players, starting with player E (where cooperation looks most unlikely). For each "test" insert into the Folk theorem formula $\frac{\varphi - \pi(c,c)}{\pi(c,c) - \pi(n,n)}$.

For player E: $\frac{155-125}{125-100} = 1.2 > 0.952$ ==> do not cooperate (which suffices to show that noncooperation is the result, as the Folk theorem criterion only needs to be violated for one player for this to occur).

(b) Show that this game can be transformed to yield a cooperative solution.

Answer: The gains from cooperation on a whole looks promising, but before we investigate this, we need to consider how many other players who will need compensation to stay with the cooperative solution.

For player D: $\frac{150-125}{125-100} = 1.0 > 0.952 \quad \Rightarrow$ do not cooperate, i.e., needs compensation

For player C: $\frac{145-125}{125-100} = 0.8 < 0.952 \quad \Rightarrow$ cooperate, i.e., does not need compensation

For players A and B we don't need to carry out the calculations given the structure in the table (as cooperation holds for C, it also holds for A and B).

The next question is if those players who gain from cooperation, and will not cheat according to the Folk theorem, have a sufficient surplus to compensate those who will not choose cooperation given that the others cooperate. The necessary compensation to D and E is found by finding the exact level where the Folk theorem holds for these two players, which is

$$\frac{x-125}{125-100} = 0.952 \quad \Rightarrow x = (0.952 \times 25) + 125 = 148.81$$

Consequently, player E needs to be compensated at least 155 - 148.81 = 6.19 in each time period, and player D needs to be compensated at least 150-148.81 = 1.19. The total room for compensation (the surplus of the three players whose best interest is to cooperate under the Folk theorem given that all players cooperate from the noncooperative equilibrium) is 75. Therefore, the necessary compensation is possible without making these three players worse off.

(c) Suppose that each player only knows his/her own payoffs (private information). Does this change the result in (b), and in case why?

Answer: It could, for two reasons. (1) Because agents E and D will try to extract extra information rents. If this rent extraction exceeds 75 (the total surplus among the other agents, which is not known by any agent due to the private information) compensation will not take place. (2) Because it would be tempting for any agent (also players A-C) to seek to extract information rents by trying to behave like D and E.

On the other hand, the upside for all agents of cooperation are quite large, implying that the willingness to seek an agreement may be quite high. As mentioned, there is no general rule for how to achieve cooperation in such cases.

Exercise 10.2: International treaties

Suppose there are 5 possible signatories to an international treaty. The treaty contains sanctions, but payoffs depend on the number of signatories. When three or more countries have signed the treaty it comes into effect.

Strategy	Country				
- sign | # other signatories	A	B	C	D	E
0	90	95	100	55	80
1	90	95	100	60	80
2	90	85	100	70	80
3	105	102	100	105	101
4	110	112	105	110	100
- do not sign | # other signatories	A	B	C	D	E
0	100	100	100	100	100
1	110	106	100	150	140
2	105	101	100	130	115
3	98	95	95	105	98
4	92	85	85	80	85

(a) Show that the treaty will be signed by at least three countries, and hence ratified. (Hint: try to work out the order in which countries will sign, and recall that once a country has signed, that is a given for the others)

Answer: Consider C, which has nothing to gain from not cooperating. C will therefore immediately sign the agreement. Both A and B has potential gains from signing, providing that at least two other players also sign in addition to C. Therefore, they will also conditionally sign.

(b) Show that once the treaty has been ratified, the remaining four countries will also sign the treaty.

Answer: Given that there are three other signatories. The agreement comes into effect (with 3 signatories). For E it is now beneficial to sign (payoff of 101 vs. 98), so he will sign. Now, given that there are four signatories and the agreement is implemented, D will also gain from signing (payoff of 110 vs. 80).