ECN 275/375 Environmental and natural resource economics
Exercise set 8 – Eirik's suggested answers

Exercise 8.1 – Taxes or tradable permits when marginal abatement costs are uncertain

The marginal damages of emissions is given by the formula $MD(M)=2M$, where M denotes emissions. The marginal abatement cost function is $MAC(M)=A-M$, where A is a constant with uncertain value between 8 and 12.

(a) Which of the policy instruments *emission taxes* or *tradable emission permits* would you choose? Justify your answer.

Answer: The MD-curve is steeper than the MAC-curve. According to the Weitzman proposition one should therefore choose tradable permits.

(b) Given your answer in (a), what is then most important of cost and damage considerations, and how does that influence your choice of instruments? Justify your answer.

Answer: When the MD-curve is steeper than the MAC-curve it follows that marginal damages are more sensitive to changes in emissions, M, than marginal costs. It is therefore more important to remove the impact of the uncertainty of the parameter A by choosing a quantity based instrument (tradable permits) where the aggregate quota is fixed, than a price instrument (emission taxes).

(c) Suppose the true value of A is 16, while the wrongful assessment of A is 12. Find the welfare losses of this wrongful assessment. Hint: Draw the solution first, then solve mathematically.

Answer: First, solve for the expected optimal emission level, M'.by setting marginal damages equal to marginal costs:

For the expected MAC: $MD(M')=MAC_{EX}(M')\Rightarrow 2M'=12-M'\Rightarrow 2M'+M'=12\Rightarrow M'=4$, which gives the expected optimal tax rate $t'=12-M'=12-4=8$. Insert t' into the correct function for the MAC (A= 16) to find the emission level corresponding to t' wich gives $t'=8=16-M^T\Rightarrow M^T=16-8=8$. Finally, find the true optimal emission level M^*, which gives $MD(M^*)=MAC_{TR}(M^*)\Rightarrow 2M^*=16-M^*\Rightarrow 3M^*=16\Rightarrow M^*=\frac{16}{3}=5\frac{1}{3}$.

Integrate to find welfare losses. For the tax (t' = 8) integrate from the true optimum M^* to M^T:

$$DW_{tax}=\int_{M^*}^{M^T}(MD-MAC_{TR})\,dM=\int_{M^*}^{M^T}(3M-16)\,dM=\Big|_{M^*}^{M^T}(3\frac{M^2}{2}-16M)$$

$$=3(\frac{8^2}{2})-16(8)-(\frac{3}{2}(\frac{16}{3})^2-16\frac{16}{3})=96-128-\frac{3}{2}\frac{256}{9}+\frac{256}{3}=10{,}67$$

For the permit (M' = 4) integrate from the true optimum M^* to M' (skipping some steps, note reversal of evaluation (largest M^* first, and smallest M' last as $MD(M') < MAC(M')$).

$$DW_{permit}=\int_{M^*}^{M'}(MD-MAC_{TR})\,dM=\int_{M^*}^{M'}(3M-16)\,dM=\Big|_{M^*}^{M'}(3\frac{M^2}{2}-16M)=2{,}67$$

This is exactly as expected – the welfare losses of using a permit is less than using a tax when the marginal damage curve is steeper than the marginal abatement cost curve.

(d) Suppose that current emissions without any regulations are 40. What is then the size of the uncertain parameter A?

Answer: A pre-regulatory emission level of 40 (= M_0) gives $0=MAC(M_0)=A-\frac{M_0}{4}=A-\frac{40}{4}$ which gives A = 10.

Exercise 8.2 – Nonmeasurable emissions

In some cases emissions are not measurable. If pollution is closely linked to the produced quantity, a way to circumvent this problem is to put a tax on production (or the consumption) of the good. There are several ways of dealing with this issue. This exercise show that with a connection between production and pollution, this approach works quite well (some tedious steps ahead, but you will learn new stuff or have something repeated).

Consider the following farm firm with the following total cost function $c(y)=a\,y+b\,y^2$, where y is the produced quantity, and the constants a and b are positive.

(a) Write down the firm's profit function.

Answer: $\pi(y)=p\,y-(a\,y+b\,y^2)$ where p is the product price such that total revenues equal py

(b) Show that the firm's profit maximizing output $y^*=\frac{p-a}{2b}$, and calculate the firm's profits:

Answer: Differentiate the profit function with respect to the firm's choice variable y, which gives the following first order condition (FOC) for a profit max:

$$\frac{\partial\pi(y)}{\partial y}=p-(a+2by)=0\Rightarrow -2by=a-p\Rightarrow y=\frac{p-a}{2b}$$

A check of the second order condition: $\frac{\partial^2\pi(y)}{\partial y^2}=-2b<0$, which is OK.

Insert the profit maximizing solution into the profit equation to get

$$\pi(y^*)=p\left(\frac{p-a}{2b}\right)-\left(a\frac{p-a}{2b}+b\left(\frac{p-a}{2b}\right)^2\right)=\frac{p-a}{2b}\left(p-a-\frac{p-a}{2}\right)=\frac{p-a}{2b}\left(p-a-\frac{p-a}{2}\right)=\frac{(p-a)^2}{4b}$$

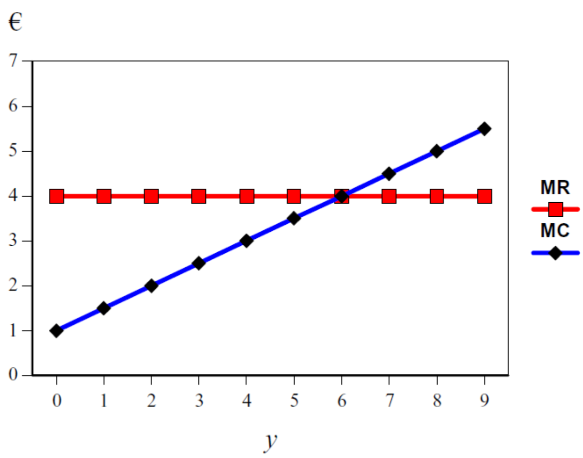
(c) Let a = 1, b = 0.25, and p = 4. Draw the firm's marginal revenue (MR) function and marginal cost (MC) functions in the same graph. From your graph, what appears to be the profit maximizing production level, y^*.

Answer: Set up the expressions for total revenues and costs to get marginal revenues and costs:

$$TR(y)=py\Rightarrow\frac{\partial TR(y)}{\partial y}=MR(y)=p \text{ and}$$

$$TC(y)=ay+by^2\Rightarrow\frac{\partial TC(y)}{\partial y}=MC(y)=a+\frac{b}{2}y$$

Inserting parameter values gives the graph to the right, and the apparent optimal solution y^* = 6 (which corresponds to the mathematical solution when inserting the values of a, b and p into the expression for y^*.



(d) The firms emissions, m, is connected to output by the following function: $m(y)=\frac{y^2}{2}$.

Assume that emissions are taxed with the tax rate t per unit of emissions. Write down the firms revised profit function.

Answer: $\pi(y)=p\,y-(a\,y+b\,y^2)-t\frac{y^2}{2}$

(e) Show that the optimal production level with the tax is: $y^T=\frac{p-a}{2b+t}$

Answer: Solve the revised profit function with the tax in the same way as in (b). (or the elegant solution – rewrite the profit function to $\pi(y)=p\,y-(a\,y+\frac{2b+t}{2}y^2)$ and the term $\frac{2b+t}{2}$ is recognizable as the term b in the solution to part (b): $y^*=\frac{p-a}{2b}$

(f) Use the parameter values from part c, and let the tax rate, $t = 0.5$. Find the firms profit maximizing output level and profits.

Answer: Insert the parameters into the y^T to get: $y^T=\frac{p-a}{2b+t}=\frac{4-1}{2(0{,}25)+0{,}5}=3$

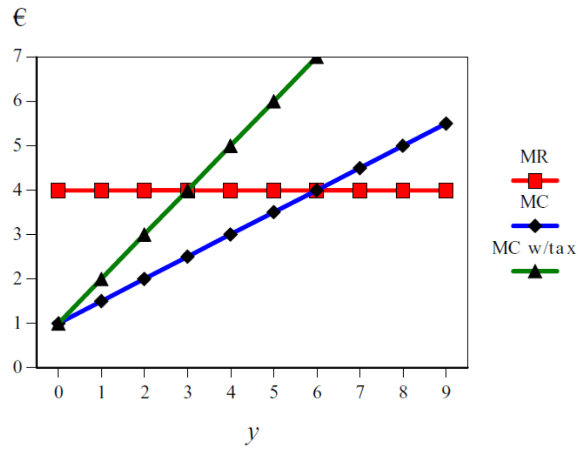
Profits: Insert the value for y^T to into your answer in (d) to get: $\pi(y^T)=2{,}25$

(g) Show the solution of the above question graphically. From the graph, how can you see that profits must decline as a result of the tax t and the reduced production ($y^T < y^*$)?

Answer: The marginal revenue function is the same as in question (c). Differentiate the revised total cost function to get marginal revenues:

$$TC=ay+(b+\frac{t}{2})y^2 \Rightarrow MC=a+(2b+t)y$$

Profits must fall because the area under the marginal revenue (the horizontal price line) and above the marginal cost curve (green line) with the tax is smaller than without the tax (blue line).



(h) Suppose that instead of a tax, the regulatory agency issues a constraint on how much the firm can produce, $y_{max} = 7$. What happens to the firms profits?

Answer: Nothing as $y_{max} = 7 \geq 6 = y^*$, the firm's pre-regulation profit maximizing quantity. Comment: non-binding constraints do not influence the optimal solution.

(i) Suppose the constraint is tightened to $y_{max} = 3$, the same output level as under the tax. Why would the firm's owner prefer this solution to the tax in the short run, but possibly not in the long run?

Answer: Because in the short run (assuming no product price change) the output under the tax and the permit are the same ($y = 3$). Under the quota the firm does not have to pay the tax, i.e., its marginal costs remain the same (the blue line).

In the long run this preference may change. A price increase makes it profitable to increase production, which a tax allows for, and the production quota does not. As such permits provide more control over emission levels than taxes, but at a possible high cost to the firms (society).