ECN 275/375 Environmental and natural resource economics
Exercise set 4 – Eirik's suggested answers

Exercise 4.1 – Benefits and damages from emissions reductions

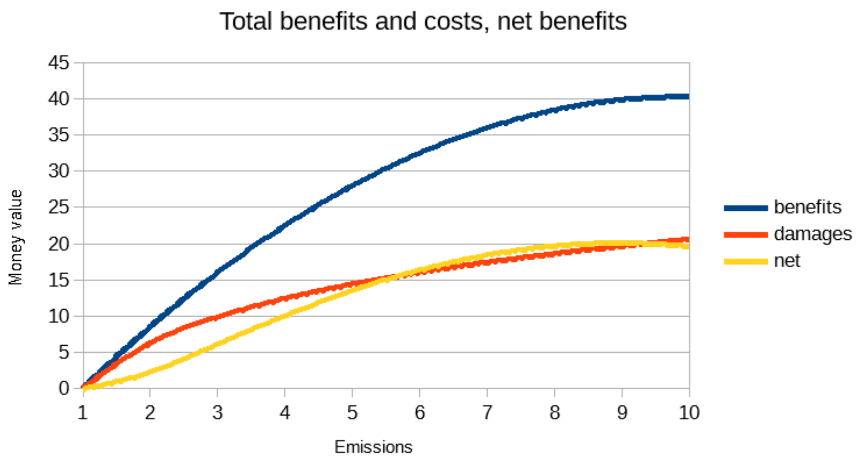
Let M denote flow emissions in tons. Total benefits from emissions in an economy are given by: $B(M)=-\frac{1}{2}M^2+10M-9{,}5$ while total damages are $D(M)=9\ln(M)$ in monetary terms.

(a) Find the marginal benefit and marginal damage functions.

Answer: Marg. benefits $MB(M) = B'(M) = 10 - M$. Marg. damages $MD(M)=D'(M)=\frac{9}{M}$

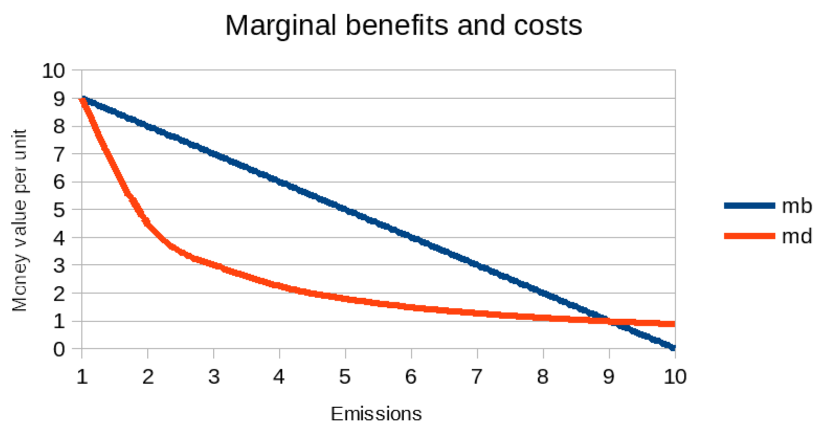
(b) Graph total benefits, total damages, and net benefits in the same figure for emissions $1 \leq M \leq 10$. What appears to be the optimal emission level.

Answer: The optimal emission level appears to be around 9 (net benefit curve is at its max).



(c) Graph marginal benefits and damages for emissions emissions $1 \leq M \leq 10$. What is now the optimal emission level.

Answer: The MB-curve crosses the MD-curve from above (at M = 9, the optimum)



Remark: it is not necessary for an optimum that MB is upward sloping, only that it crosses MD from above.

(d) Verify this by solving for the optimal emission level.

Answer: Set the marginal damage and benefit functions from (a) equal to each other and solve:

$$D'(M)=10-M=\frac{9}{M}=B'D \Rightarrow -M^2+10M-9 \text{ (2}^{nd}\text{ order polynomial } x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\text{)}$$

$$M=\frac{-10\pm\sqrt{(-10)^2-4(-1)(-9)}}{2(-1)}=\frac{-10\pm\sqrt{100-36}}{-2}=\frac{-10\pm\sqrt{64}}{-2}\Rightarrow M=0{,}5 \vee M=9$$

As benefits are non-negative, and the benefit function $B(M) = \ln(M)$ is negative for $M < 1$, it follows that the solution is $M = 9$.

Remark: Even with these simple functions and choice of parameters that made the solution come out without decimals (or fractions), things get messy quite fast. Relax – exams are testing your econ skills, not your math skills.

Exercise 4.2 – Investment in abatement technology

A firm has the following marginal abatement function: $MAC_1(m) = 10 - m$ where m denotes yearly emissions. Assume that marginal abatement costs cannot be negative.

(a) What is the firms current emissions? Justify your answer.

Answer: Emissions can be viewed as an externality. Recall that there is no "ill will" behind externalities. Therefore, the firm has no profit motivation of emitting more than 10, i.e., the firms current emissions is $m_0 = 10$.

(b) A tax on emissions is introduced with the tax rate $t_a = 3$ € per emitted unit.

Answer: Set the emission tax rate equal to the MAC function and solve (you know this already from your intro env.econ classes): $t_a = 3 = 10 - m$ ==> $m' = 7$.

Suppose a new abatement technology becomes available, so that $MAC_2(m) = 5 - m/2$.

To use the new technology the firm needs to invest 100 €. For simplicity assume the lifetime of the technology is infinite (no new technologies that is better are foreseen), and that the real interest rate, r, is 5%.

(c) Suppose that the firm chooses to adopt the new technology. What would the firm's new emission level be with the emission tax rate $t_a = 3$ € per emitted unit?

Answer: Repeat the exercise from (b) to find $m'' = 4$.

(d) Does it pay for the firm to adopt this new technology? (justify your answer)

Answer: If in doubt about the limits of the integrals, draw a figure (you find one on the next page). Remark: A figure often helps to structure the problem, and in this case also answers a-c.

Calculate the cost savings, which equal reduced abatement costs and reduced tax payments.

Difference in yearly abatement costs:

$$\Delta c_{abate}=\int_{m'}^{10}(10-m)\,dm-\int_{m''}^{10}\left(5-\frac{m}{2}\right)dm=\Big|_{m'}^{10}\left(10m-\frac{m^2}{2}+c\right)-\Big|_{m''}^{10}\left(5m-\frac{m^2}{4}+c\right) \text{ which gives}$$

when numbers are inserted -4,5, i.e., abatement costs increases as a result of the investment (the figure on the next page explains how this can happen <== abatement increases).

Difference in yearly tax payments: $\Delta c_{tax} = t_a(m' - m'') = 3(7-4) = 9$.

Total yearly cost savings $\Delta c = \Delta c_{abate} + \Delta c_{tax} = -4{,}5 + 9 = 4{,}5$, which by using the infinte time horizon formula $1/r$ for the net present value gives $\Delta c / r = 4{,}5/0{,}05 = 90$.

With an investment of 100, the total net present value is negative, and it does not pay to invest.

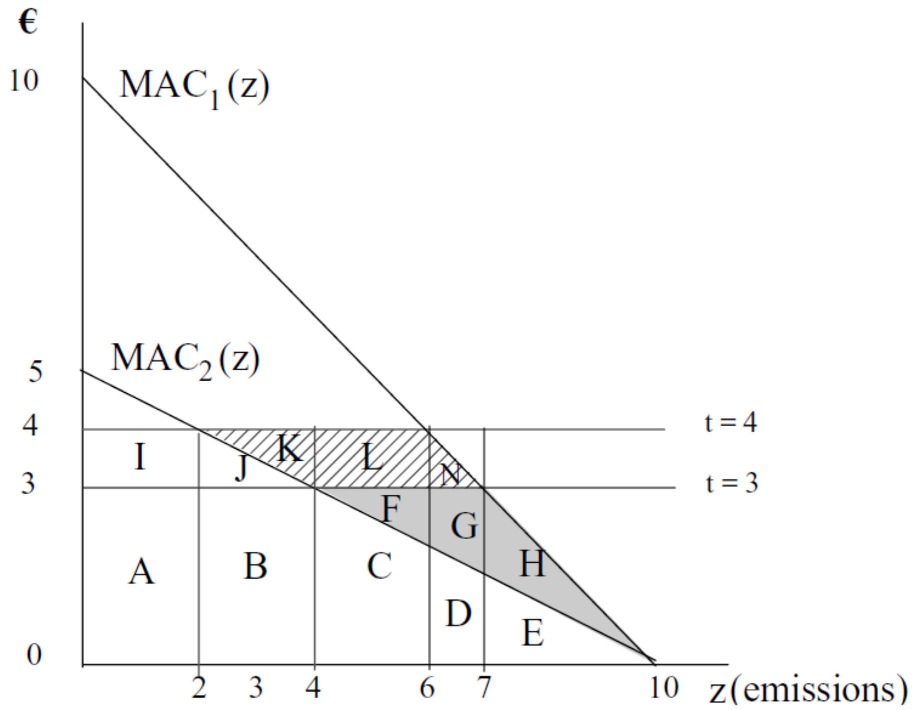
(e) Suppose the emission tax rate is increased to to $t_b = 4$ € per emitted unit? Does this change the firm's investment decision regarding the new abatement technology? If so, why?

Answer: Yearly emissions under the current and new technology are respectively $m' = 6$ and $m'' = 2$.

Insert into the modified formulas for cost savings in the previous sub-question to get yearly cost savings of the new technology: $\Delta c = \Delta c_{abate} + \Delta c_{tax} = -8 + 16 = 8$, with the infinite horizon net present value of savings equal to 160. The total cost savings are larger than the investment ==> the investment is profitable.

(f) What conclusions do you make regarding the profitability in investments and emission tax rates. A simple graph may make your discussion easier to follow.

Answer: With higher emission taxes it appears that the net present value of investing in abatement technologies becomes larger. In the graph below *m* is replaced by z. The graph matches the problem.



When the tax with the tax rate of 3 is introduced (i.e., the tax rate is increased from 0, the pre tax situation), the abatement cost savings from the new technology compared to the old technology equal areas F+G+H (the light gray shaded area). When the tax is further increased to 4, there is an additional tax cost savings from the new technology equalling areas K+N+L (the lined area).

Remark: The overall savings from a new technology triggered by a tax increase (as in this exercise) generally has the shape indicated in the figure above.