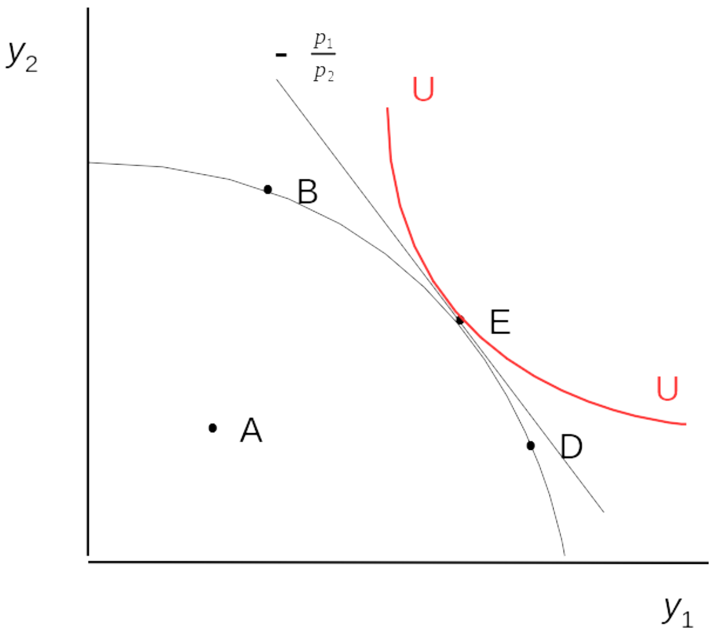
ECN 275 Environmental economics
Exercise 3 – Eirik's suggested answers

Exercise 3.1 – Technical and economic efficiency in production space

Production possibility sets define what is possible to produce with a given set of resources (expenditures or costs), C. The frontier of a production possibility set captures the trade-offs between different productions.

(a) Draw a *production possibility set* for two products y_1 and y_2. Explain briefly why the production possibility set is convex to the origin. Remember to label the axis.



Answer: The *production possibility set* (where points B, D and E are on the frontier of the set) is convex because producing more of a product causes production costs to increase (<== the convex shape of the cost function $C_i(y_i)$, where $i = \{1, 2\}$, as production increases. This implies that the slope of the marginal cost curve is positive, i.e., the second order derivative $C_i''(y_i) > 0$).

(b) In the same figure place a point for an allocation that is *technically inefficient* (point A), and two points that are *technically efficient* (points B and D). Explain why your point A is technically inefficient.

Answer: Point A is technically inefficient because for the given costs, C, it is possible to produce more of one of or both products without reducing the production of the other product, or increase the production of both products. Remark: When an allocation is *technically inefficient*, there is slack in the economy (not all resources are efficiently utilized).

(c) Add a relative price line for the two goods, and indicate given the relative price line the economically *efficient allocation*, E, of the two products.

Remark: Optimal prices p_1 and p_2 are characterized by the relative price line $-\dfrac{p_1}{p_2}$ also being tangent to the aggregate indifference curve U for the two goods y_1 and y_2. For 3 goods the relative price line turns into a plane. For 4 or more goods it turns into a hyperplane, that if optimal is a separating hyperplane between the production and consumption side of the economy.

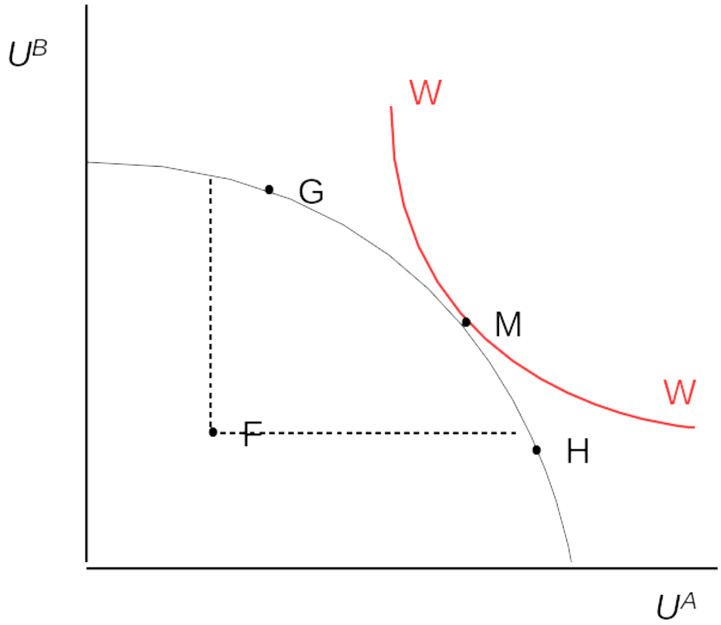
(d) Explain why point E in (c) is unlikely to be an *efficient allocation* if input or product prices do not reflect the true costs of production.

Answer: Suppose that at least one of the product prices is distorted. That results in a different slope of the relative price line $-\dfrac{p_1}{p_2}$ ==> the efficient allocation will also differ.

Exercise 3.2 – next page

Exercise 3.2 – Efficiency in consumption

Utility possibility sets depict possible combinations of utility in an economy.



(a) For a 2-person (consumers A and B) economy, draw a utility possibility set. Remember to label the axis.

Remark: the utility possibility set is usually drawn convex as a convex set relative to the origin.

(b) Insert a *Pareto-inefficient* (*-inferior*) allocation (point F) in the graph, and indicate what is the possible region of *Pareto improvements* for this set under self regarding utilities (absence of envy or "warm glow"). Explain why this allocation (F) is *Pareto inefficient*.

Remark: Under self regarding utilities, consumers only look at their own utilities and ignore the utility (well-being) of others in the economy. This gives an area north-east of the point F (and bounded by the dashed lines) which constitutes the region of *Pareto improvements*. Allocation F is *Pareto inefficient* because it is possible to increase the utility of one person without decreasing the utility of the other.

(c) Insert two allocations that are *Pareto optimal* [1] (points G and H) in the figure from (a). Explain why these points are *Pareto optimal*.

Answer: Points G and H are *Pareto efficient* because it is not possible to increase the utility of one person without decreasing the utility of the other.

(d) Draw a typical indifference curve representing society's welfare function $W(U^A, U^B)$ in the same figure. Indicate what is the *welfare maximizing* allocation (point M). Explain why this point is *welfare maximizing*.

Answer: Point M is *welfare maximizing* because it is the only allocation that given the shape of the utility possibility set and the shape of the indifference curve of the social welfare function

Remark: There is no guarantee that the market prices will produce a *welfare maximizing* allocation (M) because the social welfare function can take many different shapes depending upon voter (politician) preferences. What we know is that the resulting market equilibrium will prodcue a *Pareto optimal allocation*, i.e., an allocation on the frontier of the *utility possibility set*.

Exercise 3.3 – Welfare impacts of a reduction in uncertainty

(a) Draw a utility function (quite similar to Figure 2 on page 4 in the risk note for this lecture), with mean wealth $\bar{W}$ with equal chance of the low and high wealth allocations, W_L and W_H respectively. Indicate the welfare levels associated with the risky (low and high) wealth allocation, and the maximum welfare enhancing *cost of risk bearing* (*CORB*).

Answer: Black lines in figure on the next page relates to answer related to (a).

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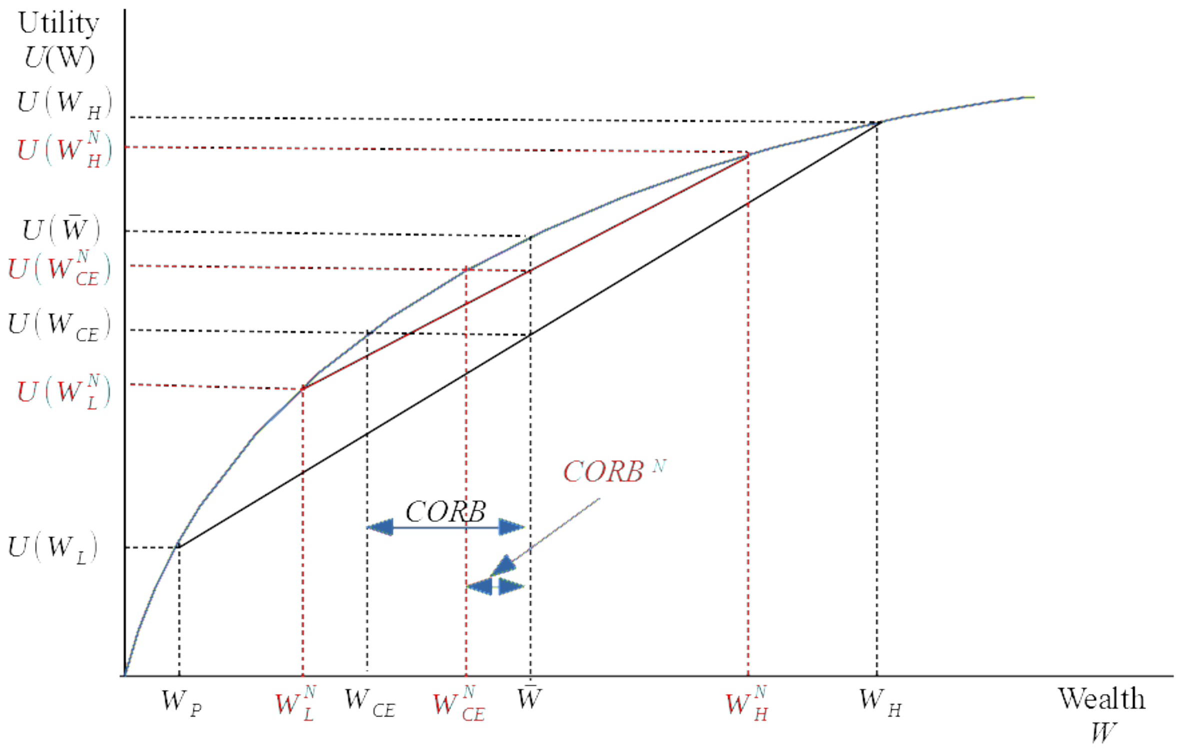
1 Alloactions that are *Pareto optimal* are also called *Pareto efficient*.



Black lines show (a) answers, and *CORB* (= the distance between mean wealth, $\bar{W}$, and the certainty equivalent W_{CE}).

(b) In figure in (a), insert a new low and high wealth allocation, W_L^N and W_H^N respectively with equal probability, and the same mean wealth, $\bar{W}$, as in (a). Indicate the difference in welfare from the risky situation in (a), and explain the reason for this difference.

Answer: Notation in red relates to the additions in (b). The difference in the utility of the two certainty equivalents equals $U(W_{CE}^N) - U(W_{CE})$. This is because the risk in the new situation is less, and for a risk averse person, this means increased utility of the risky situation.

(c) Why does the maximum welfare enhancing $CORB^N$ in (b) decline compared to (a).

Answer: The welfare loss from the risky situation is less in (b) (= $U(\bar{W}) - U(W_{CE}^N)$) than in (a) (= $U(\bar{W}) - U(W_{CE})$. Therefore, it is optimal to pay less to avert the risky situation in (b) compared to (a).

Exercise 3.4 – Quasi-option value calculation

Consider the decision tree for the quasi-option value from the risk note (Figure 5).

(a) Calculate the expected values of deciding now (commit) or delaying the decision one time period (wait) using the values: $V_0 = 20$, $V_{high} = 300$, $V_{low} = 40$, $\rho = .4$, $D_0 = 60$, and $D_1 = 120$ with no updating of the probability for a high conservation value (ρ remains at .4).

Answer: Inserting the above values into [6b] into the risk note gives:

Wait: $EV_{wait} = \rho\, MAX(V_0 + D_1, V_0 + V_{high}) + (1-\rho)\, MAX(V_0 + D_1, V_0 + V_{low})$

$$= .4\, MAX(20+120, 20+300) + (1-.4)\, MAX(20+120, 20+40)$$

$$= .4\, MAX(140, 320) + .6\, MAX(140, 60) = 128 + 84 = 212$$

Commit: $EV_{commit} = MAX\left[D_0 + D_1, \rho(V_0 + V_{high}) + (1-\rho)(V_0 + V_{low})\right]$

$$= MAX\left[60 + 120, .4(20+300) + (1-.4)(20+40)\right]$$

$$= MAX\left[180, 128 + 36\right] = 180$$

(b) What is the optimal decision, to "commit" or "wait" in time period 0? Explain why.

Answer: As the expected value of waiting (212) is higher than the expected value of committing (180), the optimal decision is to wait.

(c) Assume the optimal decision in (b) was "wait". What happens then in the next time period?

Answer: Conservation is not an irreversible decision. Hence from time period 1 and onward, there is no problem going for conservation given (a) and (b). If one later learns that the probabilities, or the values of development or conservation change in favor of development, undertaking a similar exercise again is unproblematic.