ECN 275/375 Environmental and natural resource economics
Exercise set 2

Exercise 2.1 – utility maximization

The consumer has a Cobb-Douglas utility function: $U(x_{1,}x_2)=x_1^a x_2^b$ where x_1 and x_2 are consumer goods. Let the consumer's money income be Y, and that consumer prices are p_1 for x_1, and p_2 for x_2. Assume this consumer spends all of his income Y on the two goods, x_1 and x_2. Moreover, assume that $a > 0$, $b > 0$, and that $a + b \leq 1$.

(a) What is the marginal utility of consumption for the two goods, x_1 and x_2?

(b) Show that the marginal utility of either good is positive.

(c) Show that this consumer's optimal consumption of the two goods, x_1 and x_2., equals

$$x_1^* = \frac{a}{a+b}\frac{Y}{p_1} \quad \text{and} \quad x_2^* = \frac{b}{a+b}\frac{Y}{p_2} \quad \text{respectively.}$$

(d) Explain what your solution in (c) really is.

(e) Show that consumption levels increase with increasing income, Y, and decline with increasing own prices.

Exercise 2.2 – production with mulitple inputs

A Cobb-Douglas production function with two input factors, labor (L) and capital (K) has the form $Q = A L^{\alpha} K^{\beta}$, where $A > 0$, $\alpha > 0$, $\beta > 0$, and $\alpha + \beta \leq 1$, signaling diminishing factor productivity. p is the product price for Q, w is the hourly wage, and r is the cost of capital. For a Cobb-Douglas function to be strictly positive, all inputs must be greater than zero, i.e., $L > 0$ and $K > 0$.

(a) Write down the profit function.

(b) Derive the first order conditions (FOC equations) for profit maximization.

(c) Show that the optimal ratio of the input factors equals $\frac{L}{K} = \frac{r}{w}\frac{\alpha}{\beta}$. Explain why this implies a linear expansion path?

(d) Write down the cost minimization problem for producing $\bar{Q} > 0$

(e) Solve for the cost minimizing combination of L and K. How does that relate to the answer in (c), and what is the explanation for this result?

(f) Suppose we choose to model the emissions of a pollutant, M, as an input. How would you expand your profit maximizing model formulation of the above labor-capital input model? Explain your model formulation.

Exercise 2.3 on the next page

Exercise 2.3 – discounting 1

The formula for net present value is written as $NPV(\delta)=\sum_{t=0}^{T}\frac{1}{\beta^t}(B_t-C_t)$ where $\beta = 1+\delta > 1$.

Let the duration of the project be infinite, i.e., $T = \infty$

Consider a project where investments = 10 take place in year 0 ($t = 0$) and yearly net benefits are constant in fixed monetary terms and equal to 1 (one) from year one and onwards.

(a) Calculate the net present value of this project for a 2% discount rate.

(b) Calculate the net present value of this project for a 5% discount rate.

(c) Which of the net present values is highest, and why (give an economic explanation)

Exercise 2.4 – discounting 2

In a small municipality the local politicians consider a project to make the municpality look nicer. Projected yearly benefits and costs (in 1000 NOK) for the duration (lifetime) of the project is given by the following table:

	Time (year)					
	0	1	2	3	4	5
Benefits	0	111	112	113	114	115
Costs	400	11	12	13	14	15

(a) What are the benefits of the “look nice” project with a discount rate of 9 %? Should the project be implemented at this discount rate?

(b) What are the benefits of the same project with a discount rate of 6 %? Should the project be implemented at this discount rate?

(c) Discount rates of 6 or 9 % appear quite high, in particular given the Norwegian Ministry of Finance guidelines of discount rates between 2% and 4% depending upon the type of project. Comment on the municipality’s choice of discount rate(s).