ECN 275/375 – Natural resource and environmental economics
12:15-15:15 April 10, 2024

All help aids allowed except assistance from others.
This test consists of 3 questions, for a total score of 100 points.
All questions are to be answered. You may answer in English or Norwegian.

In the case that you find a question unclear, or you are uncertain about what is meant, state the extra assumptions you need to be able to answer the question.

This test has been designed to limit the benefits of using Chat GPT and similar artificial intelligence tools. If AI use is detected beyond reasonable doubt, unreported use leads to a score of zero. Students can use AI tools if they self-report such use at a cost: A question with self-reported AI use reduces the score by 40%.

When I submit my answers on this test, I confirm that I have worked alone on my answers and not cooperated with others. I am aware that cooperation with others is to be considered an attempt or a contribution to cheat.
I am aware of the consequences of cheating (Ch. 39, Academic regulations for NMBU).

Your name: NN (+ ECN 275 or ECN 375)

Question 1 (30 points – 10 points for each sub-question a-c)

The extraction and use of non-renewable resources may involve both consumer and producer externalities.

(a) Formulate an infinite time objective function for a model with only producer externalities. State the appropriate choice variables for the long run management of a non-renewable resource, and explain what these choice variables represent as well as important parameters in this objective function.

Answer: here

(b) (i) Formulate the typical constraints needed for a model using a non-renewable resource with producer externalities caused by accumulated pollutants from the use of this resource. Extraction of the resource does not cause any immediate producer externality but contributes to the accumulation of pollution. Assume constant resource extraction costs over time. Explain the terms entering the constraints.

(ii) Express the constrained maximization problem as the current value Hamiltonian specification of the objective function with shadow prices (Lagrangian multipliers) for each of the constraints you have listed. Explain why the shadow price on the resource constraint can be replaced by the price of the resource.

Answer (i): here (constraints)

Explanation of terms in the constraints:

- text

Answer (ii): The current value Hamiltonian:

The shadow price on the resource constraint can be replaced by the market price, P_t, because … fill in …

(c) Simplify the Hamiltonian you formulated in (b) to only contain the resource constraint, i.e. as $H = U(C_t) + P_t(-R_t)$. (i) Explain why this simplification is problematic?

(ii) Suppose that this simplification is unproblematic. Draw a "four corners" graph that captures the effects of this simplified Hamiltonian when a backstop technology arrives rendering extraction of fossil resources, R_t, obsolete as the new technology completely replaces consumption of the resource. Assume that the backstop technology arrives with certainty at time T with a certain price P^B for the resource substitute. Explain your reasoning behind the graph.

Answer (i): place graph here

Answer (ii): here

Question 2 (30 points – 10 points for each sub-question a-c)

Optimal forest rotation lengths are important for the management of even aged forest stands which are conducive for area clear cuts. The basic model is the single rotation model, which will be the focus of this question unless otherwise indicated.

(a) (i) Formulate a model for management of a single rotation even aged forest stand when thinning takes place M years after the forest stand has been planted where $0 < M < T^M$, where T^M is the optimal rotation age under thinning. The cost of planting is C_0, the cost of thinning is C_M, and thinning occurs once during the forest rotation. Thinning leads to higher quality timber which triggers a price growth rate $\mu > 0$ compared to no thinning for the length of the rotation. Assume *constant net prices*, P_t, i.e., real prices per cubic meter of timber after subtracting the per cubic meter harvesting costs do not change.

(ii) Show that when thinning increases the timber price with the rate $\mu > 0$ over time gives an optimal rotation age given formula: $\dot{S}_{T^A} / S_{T^A} = r - \mu$, where r is the interest rate.

Answer (i): The objective function becomes:

Answer (ii): Showing that the optimal rotation age is given by $\dot{S}_{T^A} / S_{T^A} = r - \mu$:

(b) (i) Show that the condition for thinning to be profitable for forest owners is:
$P S_{T^M} e^{(\mu - r) T^M} - P S_{T^0} e^{-r T^0} > C_M e^{-\mu M}$ where T^M is the rotation age with thinning and T^0 is the rotation age without thinning.

(ii) Suppose that thinning is not profitable for the forest owner but marginally beneficial for society. In principle, how could forest owners be induced to conduct thinning, and should such measures be implemented? Explain the reasoning behind your answer.

Answer (i): here

Answer (ii): here

(c) The single rotation model is attractive because of its simplicity, but this simplicity comes at the costs of leaving out some aspects that could be important for optimal forest management. (i) What characterizes the aspects lost? One or two examples may be useful to illustrate your point.

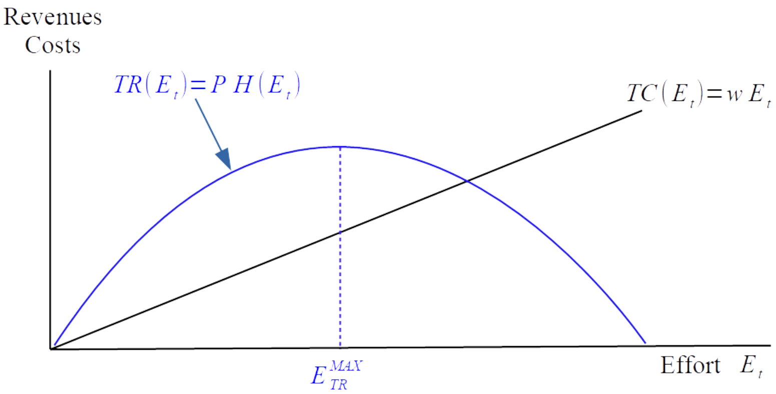
(ii) How could these lost aspects be incorporated?

Answer (i): here

Answer (ii): here

Question 3 (40 points – 10 points for each sub-question a-d)

The basic fishery effort model is often written as $\pi_t(E_t) = P\,H(E_t) - w\,E_t$, where E_t is effort, *P* is the wholesale market price for fish, $H(E_t)$ is harvest as a function of effort, and *w* is the unit costs (wage) of effort. The figure below illustrates the model.



(a) Suppose that the real price of fish, *P/w,* increases over time, i.e., *kP/w* > *P/w* when *k* > 1.

(i) Show mathematically that this leads to increased optimal effort, E_t^*, when rents (profits) from the fishery are maximized and $E_t^* < E_{TR}^{MAX}$ where .

(ii) Explain why the harvest also increases as long as $E_t^* < E_{TR}^{MAX}$ where E_{TR}^{MAX} is the effort that maximizes revenues.

Answer (i): here

Answer (ii): here

(b) (i) Draw a graph of the net growth of the fish population as a function of the stock level, $G(S)$, and explain why the instability of the equilibrium solution $\{H_t^*, S_U\}$ (where H_t^* is the profit maximizing harvest at time *t* from the harvest-effort model in (a)) requires good knowledge of the stock-net growth function $G(S)$.

(ii) In your graph also draw a second harvest level $H_t^2 > H_t^*$. Explain why this makes it more important with good estimates of $G(S)$ and to monitor the status of the fish population more accurately.

Answer (i): The net growth-fish stock graph:

Answer (ii): here

(c) At H_t^* suppose that $G'(S_U) < r$, where *r* is the risk-free interest rate. What is the rent maximizing strategy for society? Briefly explain the reasoning behind your answer.

Answer: here

(d) Show mathematically how a tax rate, $0 < T < 1$ on harvested fish can induce fishermen to lower their effort and hence the harvest H_t^* from the unstable equilibrium $\{H_t^*, S_U\}$.

Answer: here