ECN 275/375 – Natural resource and environmental economics
12:15-15:15 April 10, 2024

All help aids allowed except assistance from others.
This test consists of 3 questions, for a total score of 100 points.
All questions are to be answered. You may answer in English or Norwegian.

In the case that you find a question unclear, or you are uncertain about what is meant, state the extra assumptions you need to be able to answer the question.

This test has been designed to limit the benefits of using Chat GPT and similar artificial intelligence tools. If AI use is detected beyond reasonable doubt, unreported use leads to a score of zero. Students can use AI tools if they self-report such use at a cost: A question with self-reported AI use reduces the score by 40%.

When I submit my answers on this test, I confirm that I have worked alone on my answers and not cooperated with others. I am aware that cooperation with others is to be considered an attempt or a contribution to cheat.
I am aware of the consequences of cheating (Ch. 39, Academic regulations for NMBU).

Your name: NN (+ ECN 275 or ECN 375)

Question 1 (30 points – 10 points for each sub-question a-c)

The extraction and use of non-renewable resources may involve both consumer and producer externalities.

(a) Formulate an infinite time objective function for a model with only producer externalities. State the appropriate choice variables for the long run management of a non-renewable resource, and explain what these choice variables represent as well as important parameters in this objective function.

Answer: $\underset{\{C_t, R_t, V_t\}}{MAX} W = \underset{\{C_t, R_t, V_t\}}{MAX} \int_0^\infty U(C_t) e^{-rt} dt$

The choice variables are total consumption (C_t), resource extraction (R_t), and expenditures (V_t) for reducing the (production) externality $E(A_t)$. In addition, there is the parameter r, the risk-free interest rate.

(b) (i) Formulate the typical constraints needed for a model using a non-renewable resource with producer externalities caused by accumulated pollutants from the use of this resource. Extraction of the resource does not cause any immediate producer externality but contributes to the accumulation of pollution. Assume constant resource extraction costs over time. Explain the terms entering the constraints.

(ii) Express the constrained maximization problem as the current value Hamiltonian specification of the objective function with shadow prices (Lagrangian multipliers) for each of the constraints you have listed. Explain why the shadow price on the resource constraint can be replaced by the price of the resource.

Answer (i): The typical constraints in such a model are:

$\dot{S}_t = -R_t$ (resource stock change)

$\dot{A}_t = M(R_t) - \alpha A_t - F(V_t)$ (stock pollutant accumulation change)

$\dot{K}_t = Q(K_t, R_t, E(A_t)) - C_t - V_t$ (capital change)

Explanation of terms in the constraints:

- $\dot{S}_t$ change in the stock of the resource,
- R_t resource extraction and use (once extracted, a resource is used),
- $\dot{A}_t$ time change (derivative) in the accumulated stock pollutant A_t,
- $M(R_t)$ instantaneous emissions from resource use,
- α self cleaning factor (natural decay),
- $F(V_t)$ impacts from policy on accumulated emissions for a given policy expenditure level, V_t,
- $\dot{K}_t$ production capital change,
- $Q(K_t, R_t, E(A_t))$ the value of production from man-made capital K_t resource use R_t, and the production externality $E(A_t)$.

Remark: As extraction costs are constant over time, this variable is not listed in the simplest possible version of this model setup, extraction costs are not included.

Answer (ii): The current value Hamiltonian:

$H = U(C_t)$

$+\rho_t(-R_t)$ (resource constraint)

$+\lambda_t(M(R_t) - \alpha A_t - F(V_t))$ (stock pollutant accumulation constraint)

$+\omega_t(Q(K_t, R_t, E(A_t)) - C_t - F(V_t))$ (capital change (net investment) constraint)

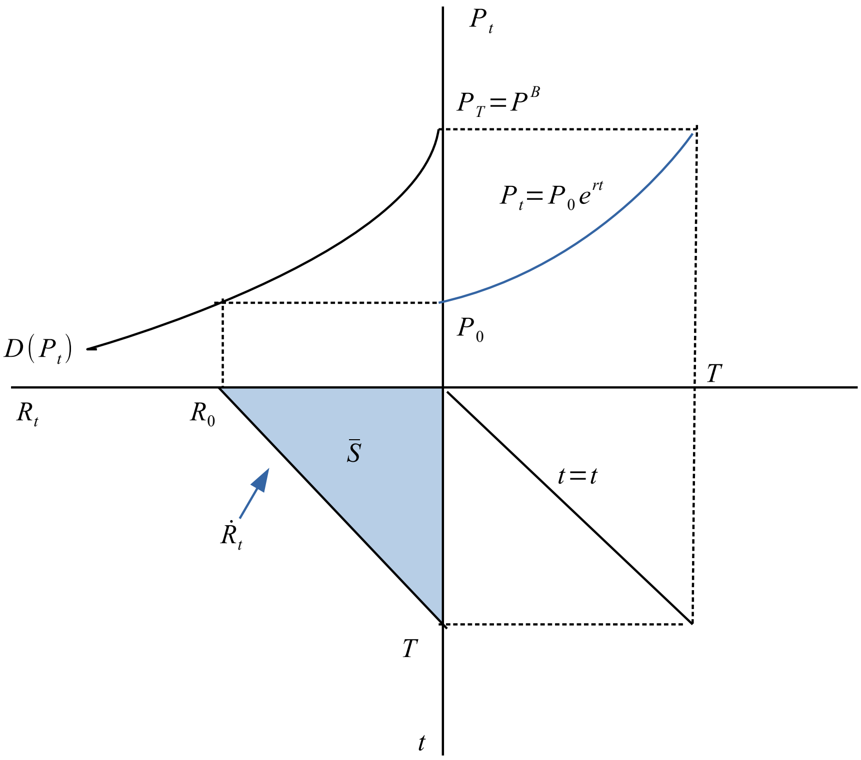
The shadow price, $\rho_t(-R_t)$, on the resource constraint $\dot{S}_t = -R_t$ can be replaced by the market price, P_t, because it represents the opportunity costs of consumption.

(c) Simplify the Hamiltonian you formulated in (b) to only contain the resource constraint, i.e. as $H = U(C_t) + P_t(-R_t)$. (i) Explain why this simplification is problematic?

(ii) Suppose that this simplification is unproblematic. Draw a “four corners” graph that captures the effects of this simplified Hamiltonian when a backstop technology arrives rendering extraction of fossil resources, R_t, obsolete as the new technology completely replaces consumption of the resource. Assume that the backstop technology arrives with certainty at time T with a certain price P^B for the resource substitute. Explain your reasoning behind the graph.

Answer (i): Even though there is no consumption externality, the production externality affects overall production negatively, which affects consumption possibilities. Hence, dropping the pollution accumulation and capital change constraints is problematic.

Answer (ii): With certain arrival of the backstop technology at time T with the certain price P^B, the “owner” of the resource will seek to empty the resource at time T. This introduces a choke price for the resource equal to the backstop price P^B for the resource substitute. Recall that the choke price is the price where the demanded quantity, here the resource, is zero. That implies $R_T = 0$, and that the demand curve meets the price axis at $P^B = P_T$.



Remark: At the time the arrival of the backstop technology and it price is known with certainty, i.e., at t=0, the initial price of the resource falls (not drawn in the graph), and the Hotelling price path follows the standard rule: $P_t = P_0 e^{rt}$.

Question 2 (30 points – 10 points for each sub-question a-c)

Optimal forest rotation lengths are important for the management of even aged forest stands which are conducive for area clear cuts. The basic model is the single rotation model, which will be the focus of this question unless otherwise indicated.

(a) (i) Formulate a model for management of a single rotation even aged forest stand when thinning takes place M years after the forest stand has been planted where $0 < M < T^M$, where T^M is the optimal rotation age under thinning. The cost of planting is C_0, the cost of thinning is C_M, and thinning occurs once during the forest rotation. Thinning leads to higher quality timber which triggers a price growth rate $\mu > 0$ compared to no thinning for the length of the rotation. Assume *constant net prices*, P_t , i.e., real prices per cubic meter of timber after subtracting the per cubic meter harvesting costs do not change.

(ii) Show that the optimal rotation period with the thinning increasing the timber price with the rate μ is given by the formula: $\dot{S}_{T^A} / S_{T^A} = r - \mu$, where r is the interest rate.

Answer (i): A constant net price allows dropping the subscript for time in the price. The objective function becomes:

$$\underset{\{T^M\}}{MAX}\, \pi(T^M) = \underset{\{T^M\}}{MAX} \left(P e^{\mu T^M} S_{T^M} e^{-r T^M} - C_M e^{-rM} - C_0 \right) \text{ where } T^M \text{ is the rotation age.}$$

Answer (ii): For more traceable notation I have replaced T^M with T . To simplify the mathematics, rewrite the objective function in the brackets to $P S_T e^{(\mu - r) T} - C_M e^{\mu M} - C_0$, set $\mu - r = -R$, and insert back in objective function to get: $P S_T e^{-RT} - C_M e^{\mu M} - C_0$.

FOC for the optimal rotation age: differentiate the objective function with *T*:

$$\frac{d}{dT}\left(PS_T e^{-RT}-C_M e^{-rM}-C_0\right)=Pe^{-RT}\frac{dS}{dT}+PS_T\frac{d}{dT}e^{-RT}=0$$

$$\Rightarrow Pe^{-RT}\frac{dS}{dT}-RPS_T e^{-rT}=0$$

$$\Rightarrow P\frac{dS}{dT}=RPS_T$$

$$\Rightarrow \dot{S}_T=RS_T$$

Substitute $\mu-r=-R$ into the above solution to get the optimal rotation age

$$T^M=T\therefore r-\mu=\frac{\dot{S}_{T^M}}{S_{T^M}}$$

(b) (i) Show that the condition for thinning to be profitable for forest owners is: $PS_{T^M}e^{(\mu-r)T^M}-PS_{T^0}e^{-rT^0}>C_M e^{-\mu M}$ where T^M is the rotation age with thinning and T^0 is the rotation age without thinning.

(ii) Suppose that thinning is not profitable for the forest owner but marginally beneficial for society. In principle, how could forest owners be induced to conduct thinning, and should such measures be implemented? Explain the reasoning behind your answer.

Answer (i): Subtract the profit equation without thinning from the profit function with thinning: $\pi(T^M)-\pi(T^0)=\left(PS_T e^{(\mu-r)T}-C_M e^{\mu M}-C_0\right)-\left(PS_{T^0}e^{-rT^0}-C_0\right)$ to get $\left(PS_T e^{(\mu-r)T}-C_M e^{\mu M}\right)-\left(PS_{T^0}e^{-rT^0}\right)>0$ for thinning to be profitable.

Answer (ii): The immediate solution is to compensate forest owners for the thinning costs at the time *M* when thinning takes place. This may be tricky as thinning costs may vary, and full compensation of thinning costs opens for higher thinning costs than what is needed.

The answer to the issue if such compensations should be implemented hinges on the size of the costs to society of inducing forest owners to thin their forest versus the benefits. All regulation entails some costs to society. With benefits being marginal, the net gains could be negative when subtracting these (administrative) costs.

(c) The single rotation model is attractive because of its simplicity, but this simplicity comes at the costs of leaving out some aspects that could be important for optimal forest management. (i) What characterizes the aspects lost? One or two examples may be useful to illustrate your point.

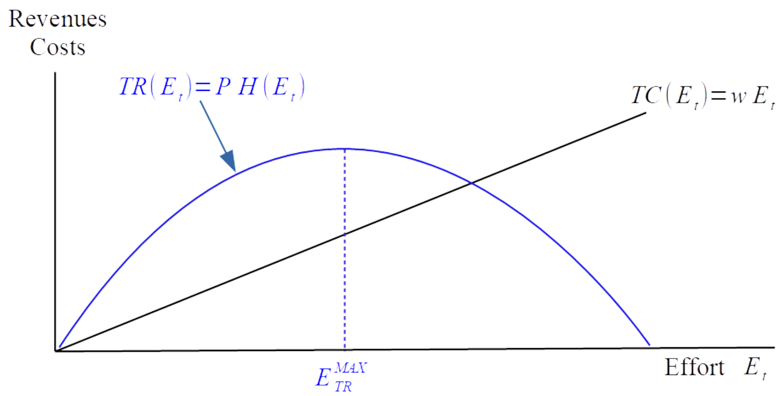
(ii) How could these lost aspects be incorporated?

Answer (i): Looking at the first order condition for the single rotation model, note that only aspects involving the optimal rotation age, *T**, are incorporated. Examples include the value of hunting occurring early in the rotation or the non-market values of increased likelihoods of seeing certain grazing animals like moose and deer when a larger fraction of the forest area is not covered with tall trees.

Answer (ii): Switching to a multiple rotation model that captures the opportunity value of land for non-timber purposes could lead to such aspects being incorporated by the forest owners. The prime example here would be hunting benefits due to a larger share of "open areas" of the total forest acreage.

Question 3 (40 points – 10 points for each sub-question a-d)

The basic fishery effort model is often written as $\pi_t(E_t) = P\,H(E_t) - w\,E_t$, where E_t is effort, *P* is the wholesale market price for fish, $H(E_t)$ is harvest as a function of effort, and *w* is the unit costs (wage) of effort. The figure below illustrates the model.



(a) Suppose that the real price of fish, *P/w,* increases over time, i.e., *kP/w* > *P/w* when *k* > 1.
(i) Show mathematically that this leads to increased optimal effort, E_t^*, when rents (profits) from the fishery are maximized and $E_t^* < E_{TR}^{MAX}$ where .

(ii) Explain why the harvest also increases as long as $E_t^* < E_{TR}^{MAX}$ where E_{TR}^{MAX} is the effort that maximizes revenues.

Answer (i): Differentiate the reformulated profit function $\pi(E_t) = kP\,H(E_t) - w\,E_t$ with effort to get $\pi'(E_t) = kP\,H'(E_t) - w = 0$ and rearrange to get $H'(E_t) = (w/kP) < (w/P)$ as *k*>0. For maximum profits $H''(E_t) < 0$. For $E_t^* < E_{TR}^{MAX}$ the optimal effort, E_t^*, must increase.

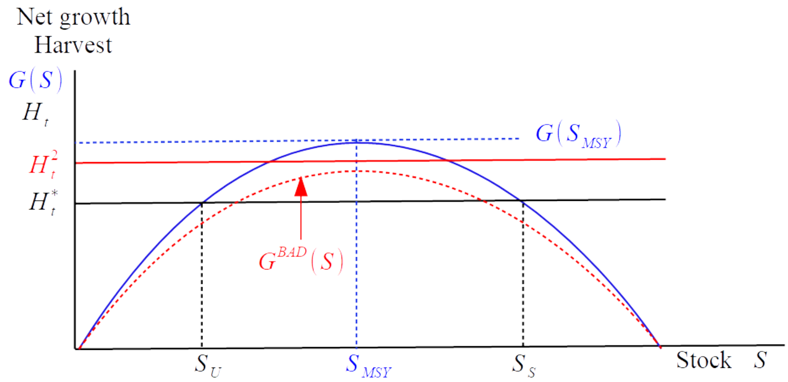
Answer (ii): The harvest increases with increasing prices when $E_t^* < E_{TR}^{MAX}$ because then $H'(E_t) > 0$ just by inspection of the above graph. Another way to show this is mathematically: the first order condition for profit maximization requires $H'(E_t) > 0$ as $H'(E_t) = (w/kP) > 0$. Note: either approach suffices for full score.

Remark: Intuitively increased prices on fish should also lead to increased harvest efforts and increased harvest (gives part score).

(b) (i) Draw a graph of the net growth of the fish population as a function of the stock level, $G(S)$, and explain why the instability of the equilibrium solution $\{H_t^*, S_U\}$ (where H_t^* is the profit maximizing harvest at time *t* from the harvest-effort model in (a)) requires good knowledge of the stock-net growth function $G(S)$.

(ii) In your graph also draw a second harvest level $H_t^2 > H_t^*$. Explain why this makes it more important with good estimates of $G(S)$ and to monitor the status of the fish population more accurately.

Answer (i): The net growth-fish stock graph.



At the harvest H_t^* there is the already mentioned unstable equilibrium $\{H_t^*, S_U\}$ and the stable equilibrium $\{H_t^*, S_S\}$ where $S_S > S_U$. For the difference $S_S - S_U$ the net income from reducing the fish stock $(P-w)(S_S - S_U)$ and collecting interest for eternity exceeds the value of the net growth of the fish stock as $r > G'(S)$ when $S > S_U$.

Answer (ii): $H_t^2 > H_t^*$ moves the harvest level closer to the maximum sustainable yield $G(S_{MSY})$. This increases the risk of over-fishing when the net growth function $G(S_t)$ is negatively affected, for example by bad growing conditions for the fish population. We could find ourselves in the following unfortunate setting:
$G(S_{MSY}) > H_t^2 > G^{BAD}(S_{MSY}) > H_t^*$, which could lead to extinction of the fish species.

(c) At H_t^* suppose that $G'(S_U) < r$, where r is the risk-free interest rate. What is the rent maximizing strategy for society? Briefly explain the reasoning behind your answer.

Answer: To lower the stock further until $G'(S_U) = r$ because at lower stock levels the derivative of the growth function increases as $G''(S) < 0$. This may come at a cost in the harvest-effort frame but that will be more than offset by the financial gains of moving some fish stock from the "fish bank" with lower returns (as $G'(S_U) < r$ without the adjustment) to the "financial bank" with returns r.

(d) Show mathematically how a tax rate, $0 < T < 1$ on harvested fish can induce fishermen to lower their effort and hence the harvest H_t^* from the unstable equilibrium $\{H_t^*, S_U\}$.

Answer: The elegant solution is to directly use your solution in (a) for the increase of a price increase with a price decrease by setting $k = 1 - T$. The less elegant (but still full score) answer is to rewrite the profit function from (a) to $\pi(E_t) = (1-T)P\,H(E_t) - w\,E_t$ which gives the solution $H'(E_t) = w/((1-T)P) > w/P$.