ECN 275/375 – Natural resource and environmental economics
12:15-15:15 April 12, 2023

All help aids allowed except assistance from others.
This test consists of 3 questions, for a total score of 100 points.
All questions are to be answered. You may answer in English or Norwegian.

In the case that you find a question unclear, or you are uncertain about what is meant, state the extra assumptions you need to be able to answer the question.

This test has been made to reduce the usefulness of ChatGPT. For this test using ChatGPT is not considered a violation of the independent work condition for tests/exams.

When I submit my answers on this test, I confirm that I have worked alone on my answers and not cooperated with others. I am aware that cooperation with others is to be considered an attempt or a contribution to cheat.
I am aware of the consequences of cheating (Ch. 39, Academic regulations for NMBU).

Your name: NN (+ ECN 275 or ECN 375)

Question 1 (30 points)

Consider the following stock pollution model for accumulated carbon A_t in the atmosphere:

$$\underset{\{C_t,R_t,V_t\}}{MAX} W = \underset{\{C_t,R_t,V_t\}}{MAX} \int_0^\infty U(C_t, E(A_t)) e^{-rt} dt$$

s.t.

$\dot{S}_t = -R_t$ (resource stock change)

$\dot{A}_t = M(R_t) - \alpha A_t - F(V_t)$ (climate gas accumulation change)

$\dot{K}_t = Q(K_t, R_t) - C_t - V_t$ (capital change)

where C_t is consumption, R_t is fossil resource use (= extraction), V_t is expenditures used to reduce environmental pressures $E(A_t)$ from accumulated emissions A_t, r is the discount rate, S_t is the stock of the resource, $M(R_t)$ is emissions from resource use, α is the self cleaning factor, $F(V_t)$ are impacts on policy for a given policy expenditure level V_t, $Q(K_t, R_t)$ is production from man-made capital K_t and fossil resource use R_t.

(a) (i) What are the signs of the partial derivatives in all functions in the above setup for the model to capture the tradeoffs between consumption C_t and accumulated emissions A_t. **(5 points)**

(ii) Suppose there is technological progress on policy impacts that grows with a rate γ. Show how this affects the relevant function(s) and alters the model setup (you only need to modify the equation(s) that are affected from this addition. **(5 points)**

Answer: (i) Partial derivatives: $U_C > 0, U_E < 0, E_A > 0, M_R > 0, F_v > 0, Q_K > 0, Q_R > 0$.

(ii) Change in policy impacts over time due to technological progress at the rate γ can be written as $F_t(V) = F_0(V) e^{\gamma t}$ which gives $\dot{F}_t(V_t) = \gamma F_0(V) e^{\gamma t} = \gamma F_t(V_t)$ when V_t is kept constant (we only capture the partial effect on the function in this time derivative).

This changes the pollution accumulation change to $\dot{A}_t = M(R_t) - \alpha A_t - (1+\gamma) F_t(V_t)$

(b) (i) How would you modify the model to include a carbon budget for staying below a certain threshold for climate change, for example the 2 degrees' threshold. **(5 points)**

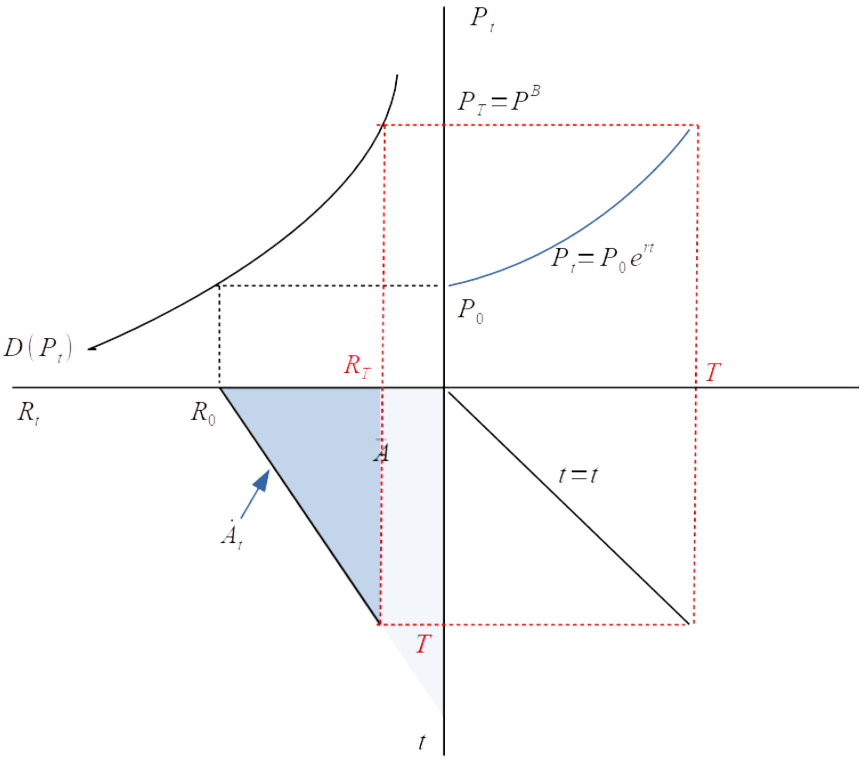
(ii) What implications emerge for the climate gas accumulation change equation in an infinite time horizon model? Explain briefly. **(5 points)**

Answer: (i) Add an extra constraint for accumulated climate gas emissions being below the threshold, here denoted $\bar{A} \geq A_t + \dot{A}_t$.

(ii) In an infinite time horizon model the climate gas accumulation change equation captured by $\dot{A}_t$ must become zero, i.e., $\dot{A}_t = 0$, before the threshold is met.

(c) Draw a four corners graph capturing the situation in (b)(i) when a backstop technology arrives rendering extraction of fossil resources, R_t , obsolete, i.e., the new technology completely replaces fossil resources. Explain your reasoning behind the graph. **(10 points)**

Answer: When constructing the light blue areas $\bar{A}$ in the third quadrant, the climate gas change equation $\dot{A}_t = M(R_t) - \alpha A_t - F(V_t)$ (or its modified version if you choose that approach: $\dot{A}_t = M(R_t) - \alpha A_t - (1+\gamma) F_t(V_t)$) gives the curvature (here a straight line for simplicity) of the total allowed accumulated climate gas amount as a function of resource use, R_t . In the graph I have ducked that problem by naming the line $\dot{A}_t$.



Remark: I have also chosen not to include any adjustment in the Hotelling price path, i.e., not adding an adjustment b to make $P_t = P_0 e^{r+b}$. In the above graph I have chosen a situation where the backstop technology arrives before the carbon budget is exceeded, i.e.,

$$\int_{t=0}^{T} \dot{A}_t \, dt = A_T < \bar{A}$$

In the opposite case that the backstop technology arrives too late for the target budget to be reached, the target must be adjusted, i.e., $\bar{A}' > \bar{A}$, to get a feasible climate policy.

Question 2 (30 points)

Optimal forest rotation lengths are important for the management of even aged forest stands which are conducive for area clear cuts. The basic model is the single rotation model. It provides many insights, but also has its limitations. Multiple rotation models cover a wider array of cases.

(a) This question relates to the single rotation version of this group of models: (i) Show that under *constant net prices*, p_t ,the optimal rotation length, T^A, is given by the formula: $\dot{S}_{T^A}/S_{T^A}=r$. (ii) Explain the intuition behind the formula given in (i). **(10 points)**

Answer:

(i) Then net price, $p_t=P_t-C_t$, where P_t is gross price, C_t is harvesting costs, and *r* is the discount rate. With a constant net price, we drop the subscript for time in the price. The objective function then becomes:

$\underset{\{T\}}{MAX}\,\pi(T)=\underset{\{T\}}{MAX}\left(pS_T e^{-rT}-k\right)$ where *T* is rotation age. FOC: differentiate with *T*

$$\frac{d}{dT}\left(pS_T e^{-rT}-k\right)=pe^{-rT}\frac{dS}{dT}+pS_T\frac{d}{dT}e^{-rT}=0$$

$$\Rightarrow pe^{-rT}\frac{dS}{dT}-rpS_T e^{-rT}=0$$

$$\Rightarrow p\frac{dS}{dT}=rp\,S_T$$

$$\Rightarrow \dot{S}_T=r\,S_T$$

The optimal rotation age $T^A=T \therefore r=\dfrac{\dot{S}_T}{S_T}$

(ii) The intuition is that the above ratio of the growth rate $\dot{S}_T$ of the standing (tree)capital S_T is the per unit (tree)value growth. This is the per unit returns of the tree(capital), which in optimum should equal the risk free returns, *r*, from ordinary (financial) capital.

(b) This question also relates to the single rotation version. Suppose there is technological progress in harvesting technologies that are used. (i) Explain how this impacts the formulation of the objective function. (ii) Explain or show how your formulation in (i) changes the optimal solution from (a), and explain the intuition behind your finding. **(10 points)**

Answer: (i) Declining timber harvest cost over time can be written as an exponential function like $C_t=C_0 e^{-\theta t}$ when $\theta>0$. When the gross price is unchanged over time, we drop the time subscript on *P*. These elements together give $p_t=P-C_0 e^{-\theta t}=p_0 e^{\gamma t}$ when $\gamma>0$.

The objective function now becomes: $\underset{\{T\}}{MAX}\,\pi(T)=\underset{\{T\}}{MAX}\left(p\,e^{\gamma T}S_T e^{-rT}-k\right)$ which simplifies further to $\pi(T)=p_0 e^{\gamma T}S_T e^{-rT}-k=p_0 e^{\gamma T}S_T e^{(\gamma-r)T}-k=p_0 S_T e^{-\phi T}-k$ where $\phi=(r-\gamma)$. This is the exact same form of the object function where $p_0=p$ and ϕ captures *r*. Differentiation therefore gives a similar result as in (a), i.e.,

$T^B=T \therefore \phi=r-\gamma=\dfrac{\dot{S}_T}{S_T}$ i.e., the rotation age increases (draw a graph if this is unclear).

(ii) Intuition. The tree capital grows at a higher rate than before due to the net price growth $p_t = p_0 e^{\gamma t}$ ==> extend the rotation age to take advantage of this until the returns on the tree capital equal *r*.

(c) Non-timber benefits. Explain why non-timber benefits occurring early in a single rotation, like hunting, do not affect the rotation age compared to the single-rotation model, but will in the multiple rotation model. **(10 points)**

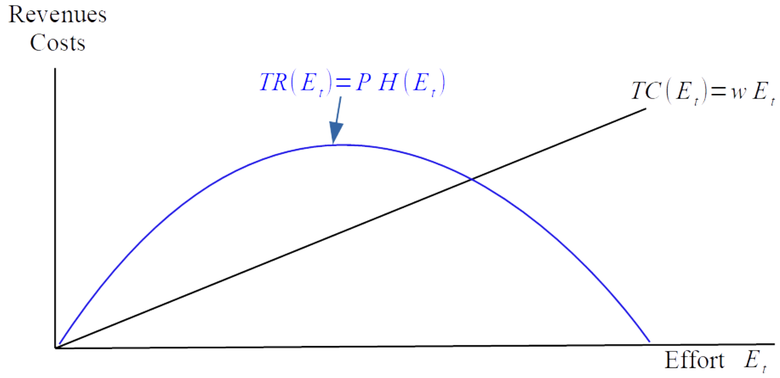
Answer: The objective function with yearly non-timber benefits (hunting) from the start of the rotation until some time $\tau < T^A$ equals $NB_\tau = \int_{t=0}^{\tau} nb_t \, dt$ with no *T* in the expression. Adding this expression to the objective function for the single rotation model in (a) will therefore not affect the optimal rotation age, i.e., $\dot{S}_{T^A} / S_{T^A} = r$ still holds.

In the multiple rotation model we know that factors like the replanting costs *k*, which contains no *T* still affects the optimal solution in the following way: negative impacts on profits like higher planting costs lead to longer rotation ages, while positive contributions like NB_τ above shortens the rotation age.

The reason for this is that increased rents early in the rotation makes it worth while to capture these rents more often. This implies shortening the rotation age in the multiple rotation models.

Question 3 (40 points)

The basic open access effort model is often written as $\pi_t(E_t) = P\,H(E_t) - w\,E_t$, where E_t is effort, *P* is the wholesale market price for fish, $H(E_t)$ is harvest as a function of effort, and *w* is the unit costs (wage) of effort. Assume that *P* is one (1). The figure below illustrates the model.



(a) Consider a "virgin" fishery, i.e., fishing occurs on a species that has previously not been fished. Such fisheries have often been unregulated. (i) Explain why the above model is well suited to describe the fishing effort in "virgin" fisheries. (ii) Explain why fisheries managed this way often lead to what is called the "tragedy of the commons". **(10 points)**

Answer: (i) High rents for the first fishing boats participating in such fisheries lead to fast and strong increases in entry which also entails high effort. The lack of regulation associated with "virgin" fisheries have often led to the open access equilibrium effort E_t^{OA} with zero rents $\pi_t(E_t^{OA}) = p\,H(E_t^{OA}) - w\,E_t^{OA} = 0$, where E_t^{OA} is the vertical line from the intersection of the TR-curve and the TC-line.

(ii) Because the high effort of open access leads to zero rents (no profits), and hence a welfare loss to society.

Remark: The high effort also implies rapid declines in the fish stocks with subsequent high possibilities that the fish becomes extinct.

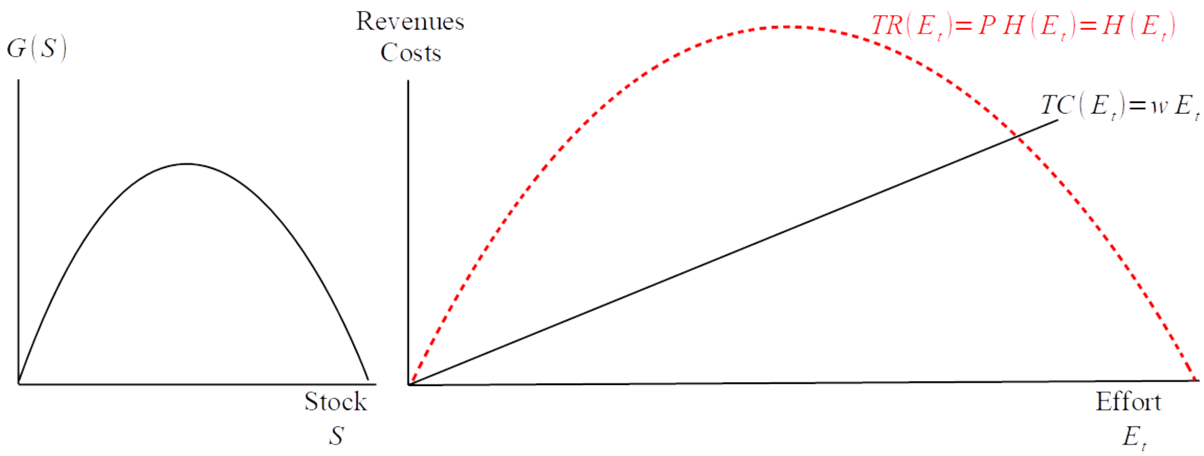
The costs per kilo fished as a function of harvests and stocks can be described by the cost function $C(H_t, S_t)$, which is increasing in harvest, i.e., $C_H(H_t, S_t)>0$, and decreasing in stock size, S_t i.e., $C_S(H_t, S_t)<0$.

(b) (i) Explain how the above information is often consistent with the open access model around the open access equilibrium effort. (ii) What modification(s) of the basic open access model $\pi_t(E_t)=p\,H(E_t)-w\,E_t$ would capture the impact of higher cost from decreased stocks, $C_S(H_t, S_t)<0$ on the fishing effort. **(10 points)**

Answer: (i) Increasing partial harvest costs $C_H(H_t, S_t)>0$ are consistent with the combined effect of the cost specification $w\,E_t$ (increasing costs in effort) in the open access model as $H_E(E_t^{OA})<0$ (negative impacts on harvest for increased effort).

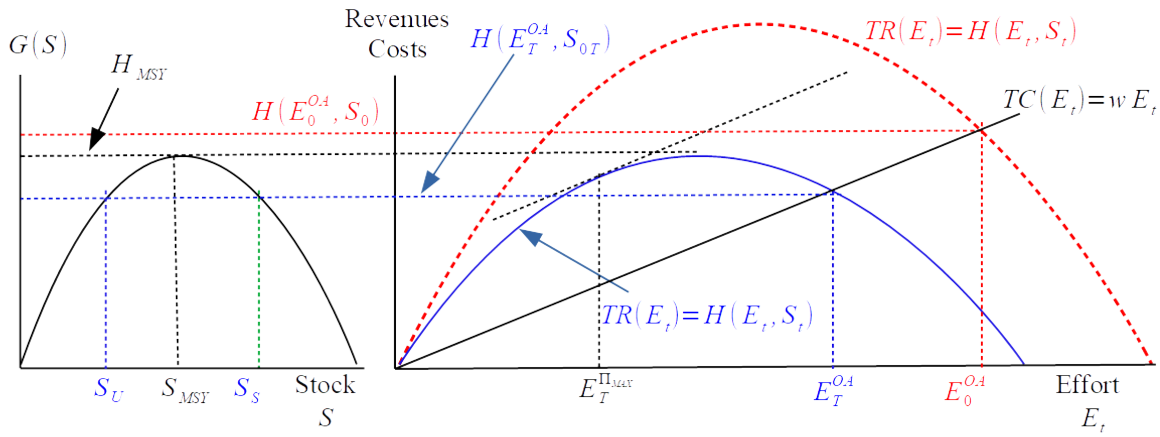
(ii) The harvest function in the basic open access model, $H(E_t)$ does not capture the partial impacts of declining stocks on harvest. Adding S_t into the harvest-effort function to get $H(E_t, S_t)$ captures this when $H_S(E_t, S_t)>0$ as lower stocks means harvests decline.

Information related to parts (c) and (d). The right hand panel in the graph below shows the harvest-effort curve (dashed red line) for a “virgin fishery”. The left hand panel of the graph shows the net growth-stock relationship (you will use that in part d). For simplicity let the product price P equals one, which gives the total revenue curve $TR(E_t)=H(E_t)$. Assume that the price $P=1$ remains unchanged over time.



(c) (i) Insert a curve in the graph above to capture the change in the total revenue curve (which with $P=1$, equals the effort-harvest curve from a “virgin fishery” (red dashed line) to an effort-harvest curve that could lead to a steady). Explain the reasoning behind the curve you added. (ii) Mark the open-access equilibria for the initial situation (dashed red line), and for the effort-harvest curve you added. Given the above graph, explain why the open access effort under the initial situation must lead to declining stocks **(10 points)**

Answer: (i) The new effort harvest curve (blue solid line in the right hand side graph) must lie inside the dashed red line as the correct answer in (b)(ii), implies that harvests, H_t, must be lower for a given effort level, E_t, under the new situation. Reason: Stocks have declined and from $H_S(E_t, S_t) > 0$ lower stocks mean harvests decline.



(ii) Open access effort levels are found where $TR(E_t) = TC(E_t)$ shown by corresponding red and blue lines and marked E_0^{OA} and E_T^{OA}. The effort E_0^{OA} leads to declining fish stocks because the harvest level $H(E_0^{OA}, S_0)$ is everywhere above the $G(S)$ function.

(d) (i) Indicate the profit maximizing effort level. Explain how you found it. (ii) Given the effort-harvest relation you drew, explain why it is sustainable or not. **(10 points)**

Answer: (i) The profit maximizing effort level is $E_T^{\Pi_{MAX}}$, and it is found where the black total cost line tangents the total revenue line for the harvest-effort function at time *T*.
(ii) The harvest that follows from $E_T^{\Pi_{MAX}}$ is sustainable because the harvest level is below $H_{MSY} = G(S_{MSY})$. Remark: Fixing the price *P* to 1 ensures that the maximum of the total revenue function equals H_{MSY} and makes it easier to link the two graphs.

Remarks: Full score is also given if $H(E_T^{\Pi_{MAX}}) > H_{MSY}$ and the fishery is managed in an unsustainable way.

In my drawing, the left panel S_U and S_S are respectively the unstable and stable equilibrium stock sizes at the harvest level $H(E_T^{OA}, S_{0T})$.