ECN 275/375 – Natural resource and environmental economics
12:15-15:15 April 6, 2022

All help aids allowed except assistance from others.
This test consists of 3 questions, for a total score of 100 points.
All questions are to be answered. You may answer in English or Norwegian.

In the case that you find a question unclear, or you are uncertain about what is meant, state your extra assumptions needed to be able to answer the question.

When I submit my answers on this exam, I confirm that I have worked alone on my answers and not cooperated with others. I am aware that cooperation with others is to be considered an attempt or a contribution to cheat.
I am aware of the consequences of cheating (Ch. 39, Academic regulations for NMBU).

Your name: NN

Question 1 (30 points)

The basic forestry model with extensions.

(a) (i) State the expression for the single rotation optimal harvest age for a basic forest model with timber benefits only. (ii) State the standard assumption(s) for this result to hold. (iii) Show how you find this expression. **(10 points)**

Answer: (i) The expression: $\frac{\dot{S}_t}{S_t}=r$

(ii) Key assumption: Net price is constant over time.

(iii) Differentiate the single rotation forest equation with the rotation age, T, and utilize the fact that $p_T = p_0$ as the net price is constant.

$$\begin{Bmatrix} MAX \\ T \end{Bmatrix} \pi(T) = \begin{Bmatrix} MAX \\ T \end{Bmatrix} p_T S_T e^{-rT} = \begin{Bmatrix} MAX \\ T \end{Bmatrix} p_0 S_T e^{-rT}$$

$$\Rightarrow \frac{\partial \pi}{\partial T} = (-r)\, p_0 S_T e^{-rT} + p_0 \dot{S}_T e^{-rT} = 0 \Rightarrow \dot{S}_T e^{-rT} = r S_T e^{-rT} \Rightarrow r = \frac{\dot{S}_T}{S_T}$$

T: The length of the optimal single rotation, also known as the harvest time.
$p_0 = p_T$: The net timber price at time T and time 0.
S_T: The timber harvest volume per hectare at time T.
r: The risk free interest rate.
e^{-rT} : The discount factor with interest r at time T.

(b) Assume that the net timber price changes over time as follows: $p_T = p_0 e^{iT}$ where i is the annual growth rate in the net timber price. (i) If the annual net price growth is positive, what impact would that intuitively have on the optimal rotation age? Explain your reasoning. (ii) How would you go about showing this mathematically? (you only need to set up the revised expression). **(10 points)**

Answer: (i) Intuitively, the rotation age should increase because including a price increase makes the the tree capital relatively more worth than financial capital ==> more investment in tree capital, which gives a higher rotation age.

(ii) The change from (a-iii) is that price growth $p_T = p_0 e^{iT}$ has been added. Recaill that in (a-iii) $p_T = p_0$.

$$\begin{Bmatrix} MAX \\ T \end{Bmatrix} \pi(T) = \begin{Bmatrix} MAX \\ T \end{Bmatrix} p_0 e^{iT} S_T e^{-rT} = \begin{Bmatrix} MAX \\ T \end{Bmatrix} p_0 S_T e^{(i-r)T}$$

$$\Rightarrow \frac{\partial \pi}{\partial T} = (i-r) p_0 S_T e^{(i-r)T} + p_0 \dot{S}_T e^{(i-r)T} = 0 \Rightarrow \dot{S}_T e^{(i-r)T} = r S_T e^{(i-r)T} \Rightarrow r - i = \frac{\dot{S}_T}{S_T}$$

$i > 0$ ==> required tree growth to maintain profits is smaller, ==> higher rotation age, *T*.

Economists tend to favor simple models as that makes it easier to get data and interpret model findings. Single rotation models are less complicated than multiple rotation models.

(c) Under what conditions do multiple rotation models provide extra insights compared to single rotation models? **(10 points)**

Answer: Multiple rotation models provide extra insights in cases where there are benefits or costs that are not captured in the equilibrium conditions for the rotation age of the corresponding single rotation model. Moose hunting is an example of such benefits that are not part of the marginal conditions for the optimal rotation age in a single period model as they are not included in the FOCs for the single period optimal rotation age. In a multiple period model this changes due to the opportunity values of land/resources.

Question 2 (30 points)

Formulate a model for maximizing social welfare with the following characteristics:

- Utility originates from consumption.
- Production takes place using natural resources, *N*, and manmade capital, *K*.
- There is a negative production externality, $E(N_t)$, from the use of natural resources. The immediate negative impacts of this externality can be reduced with the cost $V(E(N_t))$, where $V_N > 0$ and $V_{NN} > 0$.

(a) Write down the objective function, and the capital and resource constraints that correspond to the problem description. Briefly describe the reasoning behind your chosen formulations. **(10 points).**

Answer: Objective function (remark: no externality on consumption from using *N*, capital *K* is endogenous – for further explanation see captial contraint)

$$\underset{\{C_t, N_t, V_t\}}{MAX} \int_0^\infty U(C_t) e^{-rt} dt$$

with the necessary constraints:

Resource constraint: $\dot{R}_t = -N_t$
Capital constraint: $\dot{K}_t = Q(K_t, N_t, E(N_t)) - C_t - V(E(N_t))$

Remarks: Man-made capital, *K*, is endogenous in this model through the formulation of the capital constraint (Solow-type formulation), where production is negatively affected by $E(N_t)$ ($Q_E < 0$), consumption, C_t, and expenditures used to reduce the immediate production externality of natural resource use, $V(E(N_t))$.

(b) What is the interpretation of the Lagrangian multiplier for the capital constraint in this model (and for many similar style models)? Explain briefly. **(10 points)**

Answer: The value of Lagrangian multiplier for capital constraints where consumption is included (i.e., Solow-type models) equals the marginal utility of consumption. Reason: The decision maker is free to reallocate total resources into growing the capital stock or consumption. This means that in optimum the marginal value of increasing the size of the production capital must equal the marginal utility of consumption.

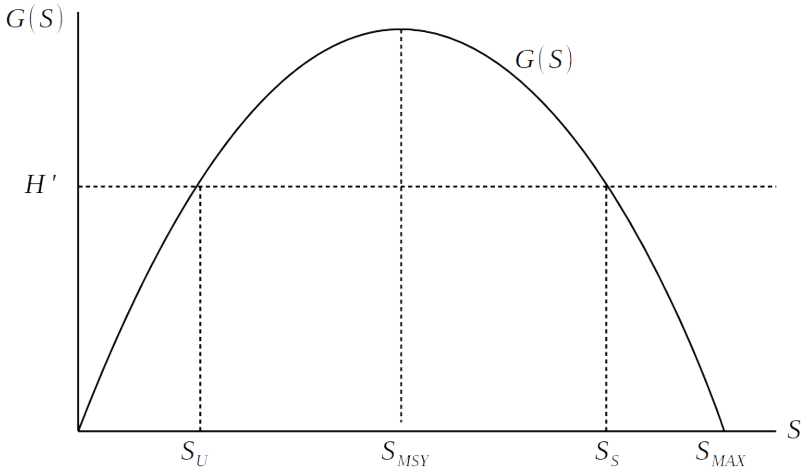
(c) Suppose that a pollutant has limited instantaneous impacts, but that its cumulative effects are large. Assume there is massive progress on the cleaning technology, leading to large expected reductions in the costs of reducing accumulated emissions. What are the effects on current and future accumulated emissions. **(10 points)**

Answer: Recall that for all types of problems with time dependence, prices and costs are to follow a Hotelling price path. With an expected fall in the future cleaning costs, the optimal strategy is to reduce cleanup efforts today, and clean up more in the future. This means allowing for an increase in accumulated emissions before the technological change has taken place, and larger declines in accumulated emissions ater the new technology is in palce. Remark: This issue was also addressed in Lecture 6 under environmental economics.

Remark: Full score also for utilizing the four quadrants graph, lowering the initial price for cleaning services, P_0 in the first quadrant to be dynamically consistent with long term expected cost savings from tech. progress, and analyze impacts on adjustment.

Question 3 (40 points)

The figure below shows a standard growth-stock size relationship for fish with the harvest level H' and two equilibrium solutions, H', S_S and H', S_U .



(a) Suppose that H' is the steady state (long run) harvest level that maximizes economic rents (profits) from this fishery. Explain why under full certainty about the stock size, S, and the growth function, $G(S)$, it is profitable to gradually adjust the fishery to the unstable equilibrium, $\{H', S_U\}$. **(10 points)**

Answer: Suppose the stock size is S_S . By fishing the fish stock $\Delta S = S_S - S_U$, the owner of the fishery (it may be a government) receives extra net revenues from the adjustment catch, $P \Delta S$ where P is the net price, and earn the capital income $r P \Delta S$. where r is the interest rate on the deposit.

(b) What is the size of the discount rate? Explain why. **(10 points)**

Answer: $r = G'(S_U)$. Reason: Indifference between fish growing in the ocean (= the fishery owner accrues wealth), or the capital gains for the profits from harvesting the fish (= the interest times the capital stock), implies that the returns of the two capital goods should be the same in equilibrium. Here, that means that the capitalization factor ((1+*r*) in discrete time) should equal the growth rate of the fish stock, i.e., $G'(S_U)$. If this was not the case, rents could be increased by adjusting the stock level so there were no arbitrage opportunities (= costless allocation change) left.

Remark: some ambiguity in the lecture notes here lead to full score for $1 + r = G'(S_U)$.

Now move to a situation that is more relevant for real life fisheries management. To keep things as simple as possible, we assume the fish lives for one year when it is mature for harvest after spawning. Suppose this is a virgin fishery, i.e., we start fishing on a newly detected fish species.

(c) At the start of the fishery for this species, we know little about the growth function, $G(S)$. Explain the learning process as the fishery proceeds. Hint: use the change in the stock and the stability properties of the equilibria to learn about the growth function. **(10 points)**

Answer: As this is a virgin fishery, we start fishing at S_{MAX} where the net growth is zero. As we gradually increase harvest levels (with some years in between without increases in the harvest, i.e., harvest is constant), we will reduce the fish stock. As long as $S_t > S_{MSY}$, we would be in a stable equilibrium, and stocks would rebound. Eventually, gradual harvest increases would lead us to a situation where $H_t > G'(S_t)$ and we would not be able to maintain our the harvest level. This is a signal that $S_t < S_{MSY}$, and that there are good reasons to reduce harvests as we have no knowledge of the shape of the growth function for $S_t < S_{MSY}$ (we have not been there yet).

(d) What are the implications of what we have learned for fisheries management under uncertainty? **(10 points)**

Answer: For this fishery with a short (one year turnaround for the fish), it may be very risky to move into $S_t < S_{MSY}$ as we now have unknown unstable equilibria ahead. Without yearly fluctuations in the growth function, we should seek a harvest level as close to but below the MSY growth rate, i.e., $H_t < G'(S_{MSY})$ to ensure we are in "stable equilibrium seas".